

ONGOING ASSESSMENT PROJECT

Ratio and Proportion Professional Development Overview

About the Ongoing Assessment Project (OGAP)

The Ongoing Assessment Project (OGAP) is a systematic and intentional formative assessment system in mathematics based on mathematics education research on how students learn specific concepts, common errors students make, and preconceptions or misconceptions that may interfere with learning new concepts or solving related problems.

The system involves using OGAP knowledge and the OGAP Frameworks/learning progressions to:

- 1) Gather evidence about pre-existing knowledge through the use of a pre-assessment;
- 2) Analyze the pre-assessment to guide unit planning; and
- 3) Implement a continuous and intentional system of instruction, probing with instructionally embedded questions, and analysis of evidence in student work to make timely instructional modifications.

Implementing the OGAP formative assessment system requires a commitment by teachers, teacher leaders, and administrators:

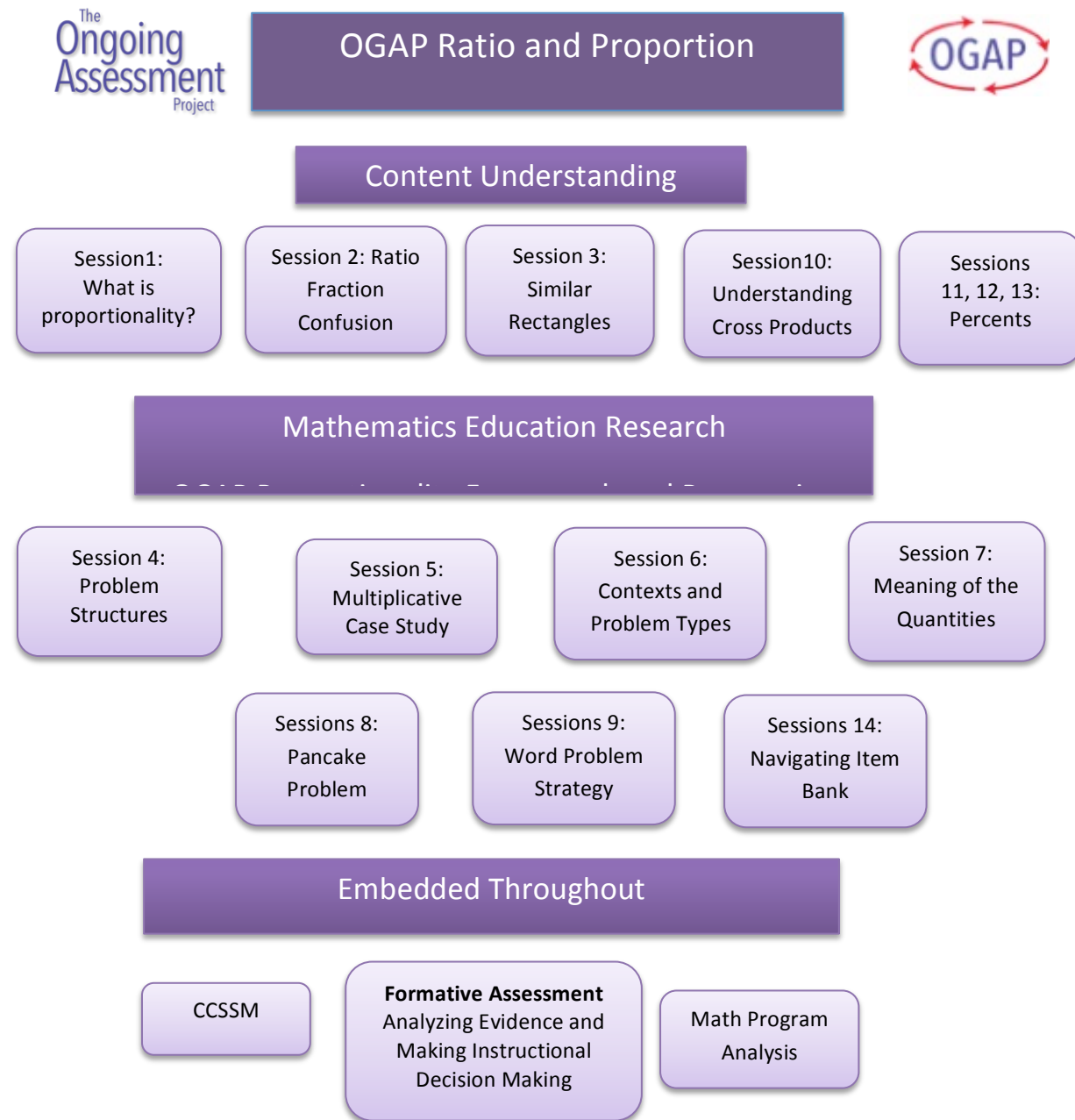
- a) To deepen professional development on related content, related math education research, and OGAP tools and strategies for gathering evidence from student work and making instructional decisions.
- b) To support and implement the ongoing nature of OGAP. For OGAP to have sustaining power districts/schools should provide teachers and teacher leaders time (e.g., regular PLC) to meet regularly to discuss evidence in student work, instructional implications, and implementation issues as they arise.
- c) To implement OGAP school wide. For example, it is strongly recommended that *all* teachers who teach mathematics (classroom teachers, special educators and interventionists), within grades 3-6 receive the professional development and ongoing support for fractions.

Evidence from OGAP work with hundreds of OGAP teachers has shown that utilizing the knowledge from the professional development is *only* solidified as teachers use this knowledge with students and that seems to happen best when teachers are supported on an ongoing basis through a system like PLCs.

IMPORTANT: *If asked to do OGAP professional development in a school or district, please secure a commitment to a system of ongoing support.*

OGAP Proportionality Professional Development Overview

OGAP Proportionality professional development consists of 16 sessions. These sessions are organized around proportionality content, mathematics education research, and evidence in student work and instructional decision-making. The sessions are shown below and on page 2 with descriptions of each session following on pages 7-14.



OGAP Proportionality Professional Development Overview

OGAP Proportionality Workshop Sessions
Session 0: Overview
Session 1: Proportional Situations
Session 2: Ratio fraction confusion
Session 3: Similar rectangles
Session 4: Problem Structures Case Study
Session 5: Multiplicative Case Study
Session 6A: Context and Problem Type Structures
Session 6B: Problem Type Case Study
Session 7: Meaning of the Quantities
Session 8: The Pancake Problem
Session 9: Word Problems
Session 10: Understanding Cross Products
Session 11: Percent estimation warm-up
Session 12: Percent Understanding
Session 13: Understanding percent increase and decrease
Session 14: Proportionality Item Bank
Other Important Documents: <ul style="list-style-type: none">• OGAP Proportionality Framework/Learning Progression• Evidence Collection documents• OGAP Item Bank• Pre-assessments• Training Items

These sessions can be completed in about 4-5 days of professional development time implemented in a number of ways: 1) as a 4-5 day workshop or course; 2) as one 2-day and one 3-day workshop or course; 3) spread throughout in-service days or a course during the school year; and, 4) other variations that maintain the integrity of the materials and the ongoing use of OGAP materials and resources.

When you are thinking about the time you have for the workshops you should consider the following important features of OGAP Professional Development remembering that the professional development supports the implementation of the OGAP Formative Assessment system.

- 1) **Design of the sessions:** The OGAP proportionality sessions intertwines math content, the mathematics education research about student learning of the concepts, analysis of evidence in student work with instructional implications, and review of instructional materials. Maintaining the integrity of these parts is critical. For example, looking at student work without understanding the research and content that underpins the work has been shown to be counterproductive. Think about each session as a package that moves participants through content, research and then implications for evidence in student work and instruction.

OGAP Proportionality Professional Development Overview

- 2) **Thinking about order and implementation:** The sessions and the parts within sessions do not necessarily have to be implemented in the order of the session numbers. The facilitator notes and the information in this document makes suggestions of order in which the sessions should be implemented and ways in which the “parts” of the sessions can be completed during separate smaller workshop or during PLC time.
- 3) **Estimated times:** The facilitator notes provide times for each session. *These times are estimates.* We have found that the actual time to implement a session is dependent upon a number of factors: size of the group, the grades the teachers teach, math content knowledge, and knowledge of curriculum and instructional strategies. Use your judgment given the situation.
- 4) **Training items and pre-assessments:** A key principle of OGAP is gathering evidence about student learning before instruction begins. To accomplish this, there are Training Items that participants should administer to their students prior to the OGAP training. The evidence from the Training Items will be analyzed as the workshop progresses. Alternately, teachers can administer the pre-assessment for the grade level they teach. These can be analyzed during the workshop or at a PLC after the training. In either case, participants will have gathered initial information about their students learning during the professional development sessions. You will find some overlap of Training Items and Pre-assessment items.
- 5) **Analyzing evidence in student work:** The whole point of OGAP is to strengthen teachers’ ability to analyze evidence in student work to help make more effective instructional decisions. To that point almost every session involves analyzing sets of student work and/or work from participant classrooms (Training Items or OGAP pre-assessments). While you can use the sets of student work in the workshop materials, the materials have greater relevance if the work comes from the participants’ classrooms. Importantly, as teachers analyze student work we ask participants to be constantly thinking about three questions.
 - a. What do you know from the evidence in student work that can be built upon?
 - b. What issues or concerns are evidenced in student work?
 - c. What are instructional implications of the evidence in student work?
- 6) **OGAP and the CCSSM:** All OGAP materials and professional development are aligned with the CCSSM. In particular, the CCSS is highlighted in a number of sessions.
- 7) **OGAP Professional Development Instructional Strategies:** These professional development materials utilize a range of instructional strategies designed to engage all participants *in thinking about* the important aspects of the workshops. Strategies such as:
 - a. **Think, Pair, Share:** “The Think-Pair-Share strategy is designed to differentiate instruction by providing students time and structure for thinking on a given topic, enabling them to formulate individual ideas and share these ideas with a peer. This learning strategy promotes classroom participation by encouraging a high degree of pupil response, rather than using a basic recitation method in which a teacher poses a question and one student offers a response. Additionally, this strategy provides an opportunity for all students to share their thinking with at least one other student which, in turn, increases their sense of involvement in classroom learning. Think-Pair-Share can also be used as an information assessment tool; as students discuss their ideas, the teacher can circulate and listen to the conversations taking place and respond accordingly.” <http://www.readwritethink.org/professional-development/strategy->

OGAP Proportionality Professional Development Overview

[guides/using-think-pair-share-30626.html](https://www.ogapmath.org/guides/using-think-pair-share-30626.html) The think, pair, share strategy is used throughout OGAP training.

- b. **Group work:** We suggest that participants be in groups of not more than 3 or 4 people for the most effective use of group work.
- c. **Questioning:** Questioning is used throughout all OGAP sessions as a strategy to deepen understanding of targeted concepts and ideas. In some cases probing questions are provided. However, facilitators should not limit themselves to those questions provided if opportunities arise.
- d. **Sharing Solutions:** The point of sharing solutions is to help deepen understanding of a concept. The point is NOT to give participants an opportunity to participate. For this strategy to be effective the facilitator must carefully select solutions to share with the mathematical goal in mind. See Session 1 Facilitator Notes, page 4 of the for an example.
- e. **Poster Sessions:** The point of poster sessions is to get *all* participants to think deeply about an idea or concept. We have found that the depth of discussion and thought increases when participants have to commit their ideas to a public poster. In addition, poster sessions are designed to synthesize ideas and concepts. See Session 3 Facilitator Notes, page 9 for one example of how to debrief a poster session.
- f. **Problem Solving:** Many of the sessions start with a set of problems or a single problem for the sole purpose of engaging participants in the mathematical ideas related to the session. Examples include: Sessions 1, 4, 6, 9, 10, 11, 12, 13, and 14. Providing participants ample time to engage in the problems, activity, or sets of problems in these sessions is critical.
- g. **Formative assessment:** Every session is designed to help the facilitator gather evidence about participant learning to guide their facilitation: a) listening and observing during individual work; b) listening and observing group discussions; c) analysis of posters as they are developed and discussed; d) Full group discussion; and, e) embedding problems into instruction as needed as entry or exit cards.
- h. **Using daily workshop evaluations:** Appendix B contains a sample of a daily evaluation. The point of the evaluation is for you to understand what is working and what needs modification as the workshop progresses along with eliciting any questions that participants have that need clarification. We have instituted a daily protocol of opening the following day's session by reviewing the evaluation information, addressing questions and concerns, and explaining any adjustments that have been made to the workshop as a result of the feedback. We suggest that you make a three slide power point: What's working; Issues/Concerns; Questions.

It is important to be explicit as the workshop progresses about the different instructional strategies you are using; what they are and why they are important. Most importantly, be clear that these same strategies can be used when participants are working with their own students.

OGAP Proportionality Professional Development Overview

Some suggested readings about instructional strategies:

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn*. Sausalito, CA: Math Solutions.

Lamberg, T. (2013). *Whole Class Discussions: Improving In-depth Mathematical Thinking and Learning*. Pearson Publishing.

Smith M., & Stein, M.K. (2011) *5 Practices for Orchestrating Productive Mathematics Discussions*. National Council of the Teachers of Mathematics, Reston, VA.

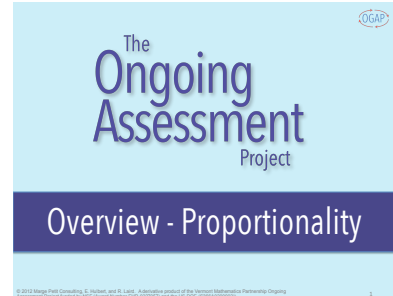
Walsh, J., & Sattes, B. (2005). *Quality Questioning: Research-based practice to engage every learner*. Corwin Press, London, England.

Wiliam, D. (2011). *Embedded Formative Assessment*. Solution Tree Press, Bloomington, IN.

OGAP Proportionality Professional Development Overview

Session 0: Introduction to OGAP

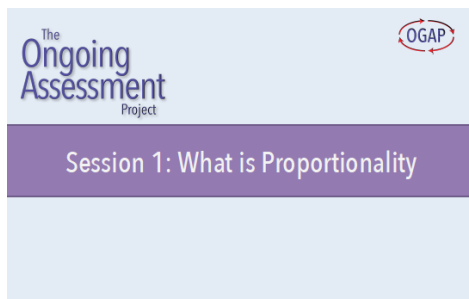
The focus of this session is to engage participants in an introduction to the Ongoing Assessment Project and in discussions about formative assessment and learning progressions.



Session Sequence:

- A) OGAP background and Guiding Principles (5 minutes)
- B) Research on Formative Assessment and Examples of Formative Assessment Strategies (5 minutes)
- C) OGAP system (5 minutes)
- D) Learning Progressions (15 – 20 minutes)
- E) Overview of OGAP Proportional Reasoning Training (5 minutes)

Session 1: What is proportionality?



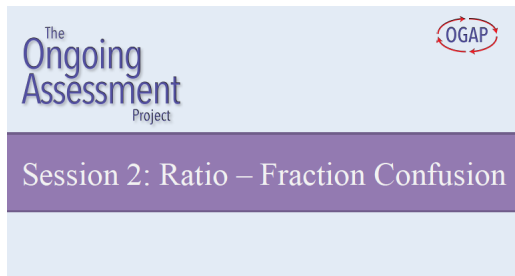
In this session participants will build understanding of the characteristics of proportional situations. Participants will complete a graphic organizer individually and as a group that is designed to gather information about examples and non-examples of proportional situations. Participants will then analyze four non-numerical graphs and select the one that represents a proportional situation. Additionally, the group will generate contexts that fit different proportional situations. As

the workshop progresses table groups will return to their graphic organizer to add, delete, or clarify ideas. This initial exercise is designed to help workshop facilitators gather information about participant understanding of proportionality.

Session Sequence:

- A) Graphic organizer (20 minutes)
- B) Graph analysis (10-15 minutes)
- C) Creating graph examples (15 -25 minutes)

OGAP Proportionality Professional Development Overview



Session 2: Ratio fraction Confusion

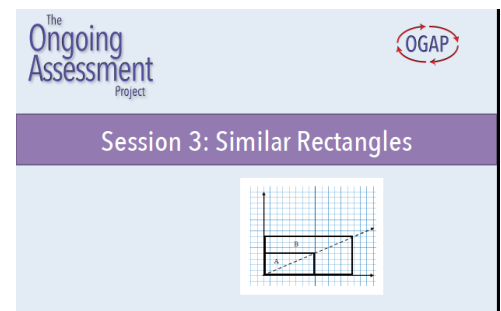
This session is designed to unravel conceptions and misconceptions about how ratios and fractions are related. Participants will solve a simple ratio problem that will serve as the platform for discussion about ratios and fractions. Then participants will analyze and exemplify situations in which ratios behave like fractions.

Session Sequence:

- A) Basketball problem (10-15 minutes)
- B) Ratio Fraction Poster (20-30 minutes)

Session 3: Similar Rectangles

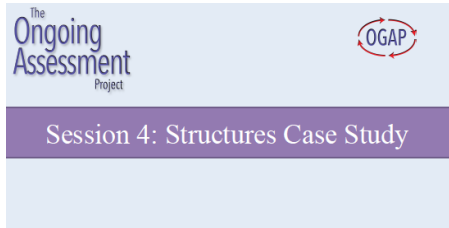
This session is a follow-up to Sessions 1 and 2. It can be implemented at any time during the workshop. Participants will analyze two graphs; one representing similar rectangles, and the other not. They will then analyze the components of the graph. That is, the line, points along the line, and the axis.



Session Sequence:

- A) Analyzing graphs
 - a. Part I - Are these graphs right? (10 minutes)
 - b. Part II – Understanding elements of the graphs (15 – 20 minutes)

OGAP Proportionality Professional Development Overview



Session 4: Structures Case Study

In this session participants first solve 8 problems and then analyze each problem for features/structures of the problems. The participants then sort the problems into three categories: easiest; moderate; most challenging. As they sort the problems participants record features that influenced their decisions.

This session and the next few sessions identify structures of problems that influence student solutions to problems involving ratios, rates, and proportions.

Research indicates that student solutions move back and forth between using proportional, transitional, and non-proportional reasoning depending on the structure of the problems, the context in which the problems are situated, and the strength of the students proportional reasoning. (Cramer, Post, & Currier, 1993, Karplus, Pulos & Stage, 1983, VMP OGAP Pilots, 2006 & 2007)

Session Sequence:

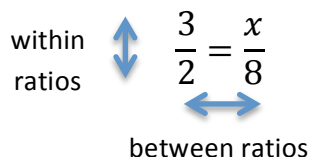
- A) Solve problems (15 minutes)
- B) Sort problems (10 minutes)
- C) Debriefing (10 minutes)

Session 5: Multiplicative Case Study

In this session participants will be introduced to the OGAP Proportionality Progression, issues related to the multiplicative relationships in proportionality problems, and then review a small sample ($n = 6$ students) of student work gathered during a 2006 OGAP study. The study was designed to explore the impact on student solutions of changing the multiplicative relationship ‘within’ and ‘between’ ratios in problems involving proportionality.



These terms, ‘within’ and ‘between’ ratios gives us language to discuss the multiplicative relationships in a proportion. The diagram illustrates what is meant by ‘within’ a ratio and ‘between’ a ratio.



Session Sequence:

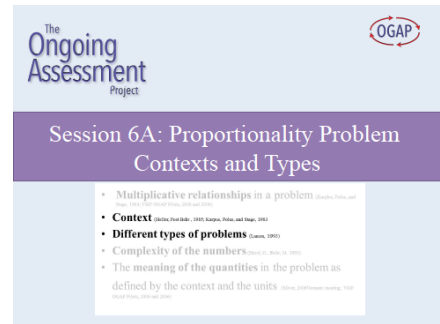
- A) Introduction to OGAP Proportionality Progression and Structures (15 minutes)
- B) The Case Study – Solving Problems (15 minutes)
- C) Analysis of Student Solutions (15 – 20 minutes)
- D) Discussion Questions (10 minutes)

OGAP Proportionality Professional Development Overview

Session 6A: Context and Problem Types

In this session participants will be introduced to the different problem types found in ratio and proportion problems. The session involves understanding the structure of the problem types and analyzing student work using the OGAP Proportionality Progression to see how the structure might impact performance.

Session Sequence: Think of this session as 6 distinct parts. Suggested BREAKS are indicated below.



Part I: Introduction to session

- A) **Context and the CCSSM** (10 - 15 minutes)
- B) **Intro to problem types and the OGAP Proportionality Progression** (3-5 minutes)

Part II: Ratios and rate definition and student work

- C) **Ratio and rates definition and introduction** (15 minutes)
- D) **Ratios and evidence in student work for ratio** (30 minutes)

BREAK

Part III: Rates

- E) **Rates and evidence in student work** (45 minutes)

Part IV: Missing value and internal structure of problems

- F) **Missing value and internal structure** (10 - 15 minutes)
- G) **Evidence in missing value problems** (40 minutes)

BREAK

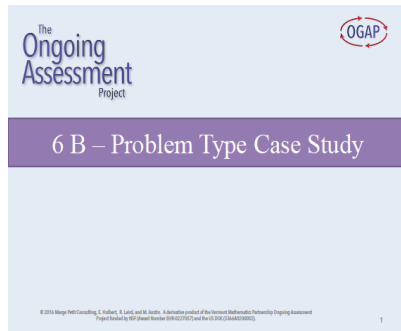
Part V: Similarity and scale factor problems

- H) **Similarity and scale factor problems** (45 minutes)

Part VI: Math program review (Optionally done during this session or after Session 6 B)

- I) **Your math program and problem structures** (20 minutes) OPTIONAL – Can be completed at the end of Session 6b or during PLCs.

OGAP Proportionality Professional Development Overview



Session 6B: Problem Type Case Study

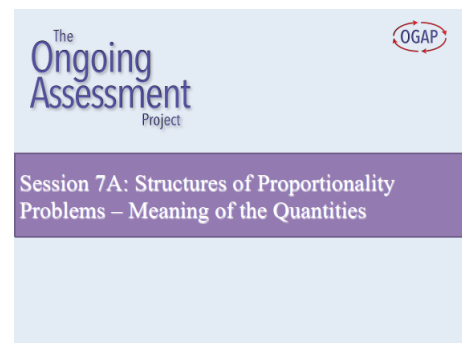
This case study is designed to help participants understand the importance of varying problem structures. In particular, it focuses on contrasting proportional and non-proportional problem structures. Participants will solve two problems; one involving a proportional situation and the other a non-proportional situation. Participants will then sort student work from an OGAP study involving 82 sixth grade students, reflect on the data from the OGAP study, and engage in a discussion about the instructional implications of the findings.

Session Sequence:

- A) Solve and debrief problems (10 - 15 minutes)
- B) Review research, conduct student work analysis (20 minutes)
- C) OPTIONAL – Review Training Items from participant classroom (15 minutes)
- D) OPTIONAL – Analyze math program for problem structures (25 minutes)

Session 7: Meaning of the Quantities

In this session participants will review 5 pieces of student work. The first four student responses are examples of the types of errors that students make when they are not considering the quantities in the problem or in the solution. Participants are asked to address two issues when analyzing student responses 1 – 4.



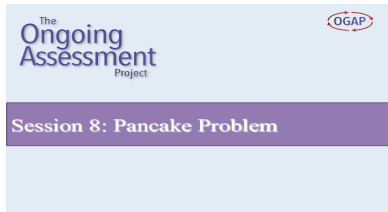
- 1) What is the evidence that the student may not be interpreting the meaning of the quantities in the problem correctly?
- 2) Suggest some questions you might ask each student or activities you might do to help them understand the meaning of the quantities in the problem and the solution.

Session Sequence:

- A) Part I – Analyzing student responses 1-4 (20-25 minutes)
- B) Part II – Evidence in Solution 5 (5-10 minutes)

OGAP Proportionality Professional Development Overview

Session 8: The Pancake Problem



In this session participants solve the Pancake problem. They place multiple solutions on chart paper, analyze the evidence in the solutions, and then analyze the problem for structures. This session can be used as a warm-up to start a day as a way to reinforce and review structures of problems and strategies on the OGAP Progression.

Session Sequence:

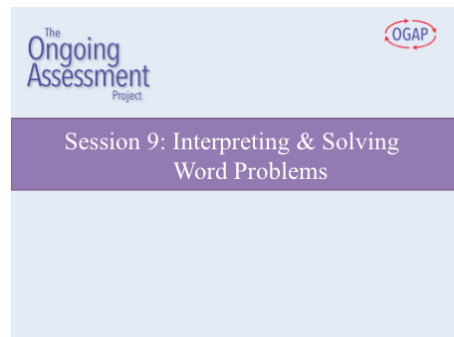
- A) Solve the problem (20-30 minutes)
- B) Problem Structures in Pancake Problem (10 minutes)
- C) Strategies and OGAP Progression (10 minutes)

Session 9: Interpreting Word Problems

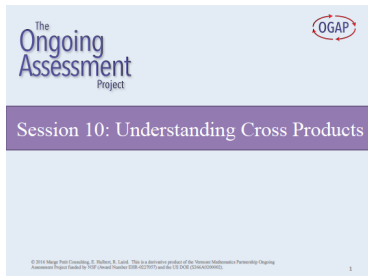
In this session participants will engage in a strategy to help students solve word problems. The strategy is adapted from researched- based literacy strategies.

Session Sequence:

- A) Interpreting and solving word problems (30 minutes)



Session 10: Understanding Cross Products



This session is designed to help participants understand the mathematics underpinning the cross products strategy. Participants will solve a problem using unit rate, factor change, and cross products strategies, compare the expressions representing these strategies, and then use the Multiplication Property of Equality to understand the cross products strategy.

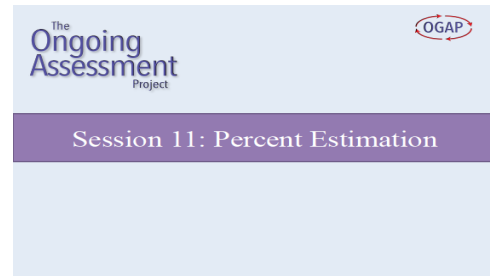
Session Sequence:

- A) Understanding Cross Products (45 minutes)

OGAP Proportionality Professional Development Overview

Session 11: Percent Estimation

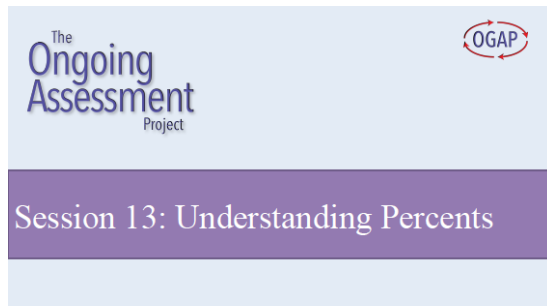
In this session participants solve percent problems using estimation and mental arithmetic strategies. They will then create a set of percent problems for their students to use as warm-ups. This session reinforces research finding that “using strategies to estimate solutions with percent can deepen understanding and flexibility when using percents.” (Hunkier, 2002)



Session Sequence:

- A) Mental arithmetic and estimation warm-up (5-10 minutes)
- B) Writing and sharing percent/decimal warm-up problems (15 – 20 minutes)

Session 12: Understanding Percents



In this session participants complete a graphic organizer and then solve a series of percent problems using 100 grids. A key understanding of percent is the idea of using an independent unit (100) to ‘norm to a standard’; in this case 100.(Lamon, 1999) The use of percents as a norming standard facilitates comparisons. Students are often taught definitions and procedures about percents with little understanding of the value of using percents.

Aspects of this session were adapted from a NCTM Illuminations Activity at <http://illuminations.nctm.org/LessonDetail.aspx?id=L249> . Permission is not required to use this activity.

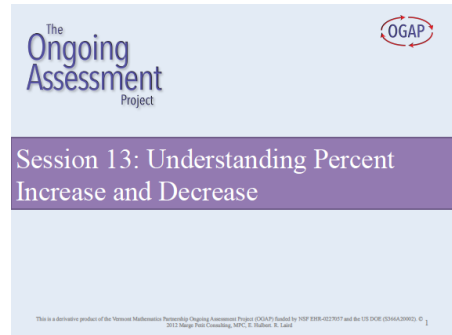
Session Sequence:

- A) Percents graphic organizer (15-20 minutes)
- B) Solving percent problems using percent grids (30 minutes)
- C) Big ideas (10 minutes)

OGAP Proportionality Professional Development Overview

Session 13: Percent Increase and Decrease

This activity is designed to engage participants in the concepts underpinning percent increase and decrease without focusing on manipulating the calculations. Percent increase and decrease are difficult concepts for students because they involve a difference quantity and additive language in a multiplicative relationship. (Edwards (1930) Smart (1980) Venezky and Bregar (1988) as noted in Parker dissertation page 25, and Guiler (1946a, 1946b) as noted in Parker and Leinhardt)



The materials for this activity are used with permission of the Mathematics Assessment Project, The Shell Center, University of Nottingham and UC Berkley

<http://map.mathshell.org.uk/materials/tasks.php?subpage=apprentice&taskid=250>

Session Sequence:

- A) Warm-up and research review (10-15 minutes)
- B) Instructions and making model (30 -40 minutes)
- C) Debriefing (15-20 minutes)

Session 14: Navigating the Item Bank

In this session participants will become familiar with the item bank organization by engaging in activities that help participants:

- Understand the purpose of the item bank.
- Understand how problem structures are reflected in the item bank.
- Learn how to access items using problem contexts, problem types, and other structure.
- Select items from the bank based on an instructional need

