About the Ongoing Assessment Project (OGAP)

The Ongoing Assessment Project (OGAP) is a systematic and intentional formative assessment system in mathematics based on mathematics education research on how students learn specific concepts, common errors students make, and preconceptions or misconceptions that may interfere with learning new concepts or solving related problems.

The system involves using OGAP knowledge and the OGAP Frameworks/learning progressions to:

1. Gather evidence about pre-existing knowledge through the use of a pre-assessment;
2. Analyze the pre-assessment to guide unit planning; and
3. Implement a continuous and intentional system of instruction, probing with instructionally embedded questions, and analysis of evidence in student work to make timely instructional modifications.

Implementing the OGAP formative assessment system requires a commitment by teachers, teacher leaders, and administrators:

a. To deep professional development on related content, related math education research, and OGAP tools and strategies for gathering evidence from student work and making instructional decisions.

b. To support and implement the ongoing nature of OGAP. For OGAP to have sustaining power districts/schools should provide teachers and teacher leaders time (e.g., regular PLC) to meet regularly to discuss evidence in student work, instructional implications, and implementation issues as they arise.

c. To implement OGAP school wide. For example, it is strongly recommended that *all* teachers who teach mathematics (classroom teachers, special educators and interventionists), within grades 3-6 receive the professional development and ongoing support for fractions.

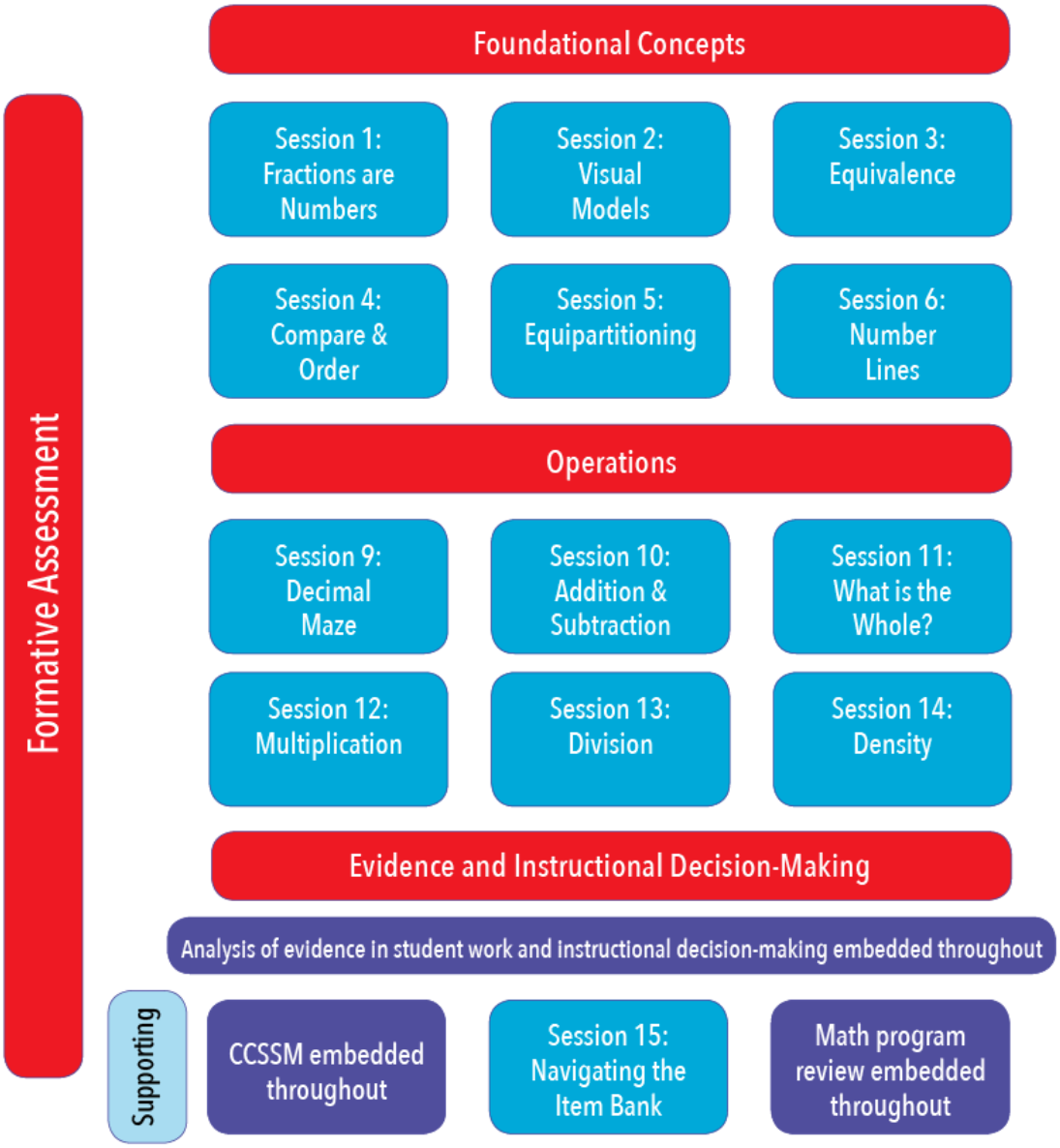
Evidence from OGAP work with hundreds of OGAP teachers has shown that utilizing the knowledge from the professional development is *only* solidified as teachers use this knowledge with students and that seems to happen best when teachers are supported on an ongoing basis through a system like PLCs.

IMPORTANT: If asked to do OGAP professional development in a school or district, please secure a commitment to a system of ongoing support.

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) funded by NSF EHR-0227057 and the US DOE (S366A20002).

About OGAP Fractions Professional Development

The OGAP Fraction professional development involves 15 sessions divided into 2 parts: foundational knowledge; and, knowledge about operations with fractions. Each session intentionally intertwines aspects of content, knowledge of mathematics education research, analysis of evidence in students work with instructional decision making, and analysis of instructional materials in light of math education research.



These sessions can be completed in about 4-5 days of professional development time implemented in a number of ways: 1) as a 4-5 day workshop or course; 2) as one 2-day and one 3-day workshop or course; 3) spread throughout in-service days or a course

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during the school year; and, 4) other variations that maintain the integrity of the materials and the ongoing use of OGAP materials and resources.

When you are thinking about the time you have for the workshops you should consider the following important features of OGAP Professional Development remembering that the professional development supports the implementation of the OGAP Formative Assessment system.

1. **Design of the sessions:** Each of the OGAP Fraction sessions intertwines math content, the mathematics education research about student learning of the concepts, analysis of evidence in student work with instructional implications, and review of instructional materials. Maintaining the integrity of these parts in critical. For example, looking at student work without understanding the research and content that underpins the work has been shown to be counterproductive. Think about each session as a package that moves participants through content, research and then implications for evidence in student work and instruction.
2. **Thinking about order and implementation:** The sessions and the parts within sessions do not necessarily have to be implemented in the order of the session numbers. The facilitator notes and the information in this document makes suggestions of order in which the sessions should be implemented and ways in which the "parts" of the sessions can be completed during separate smaller workshop or during PLC time.
3. **Prioritizing sessions for grade spans:** Which sessions you focus on with a group may depend upon the grade levels of the participants. For example, if you are only working with teachers in grades 5-7 you might include Session 1, Part I of Session 2, Session 3, Session 4, Session 6, Session 8, and all the operations sessions. On the other hand, if you are only working with grades 2-4, then your focus will be in Sessions 1 – 7 and Session 10 and Part I (A and B) and Part II of Session 11. Use your professional judgment.
4. **Estimated times:** The facilitator notes provide times for each session. *These times are estimates*. We have found that the actual time to implement a session is dependent upon a number of factors: size of the group, the grades the teachers teach, math content knowledge, and knowledge of curriculum and instructional strategies. Use your judgment given the situation.
5. **Training items and pre-assessments**: A key principle of OGAP is gathering evidence about student learning before instruction begins. To accomplish this, there are Training Items that participants should administer to their students prior to the OGAP training. The evidence from the Training Items will be analyzed as the workshop progresses. Alternately, teachers can administer the pre-assessment for the grade level they teach. These can be analyzed during the workshop or at a PLC after the training. In either case, participants will have gathered initial information about their students learning during the professional

development sessions. You will find some overlap of Training Items and Pre-assessment items.

6. **Analyzing evidence in student work:** The whole point of OGAP is to strengthen teachers' ability to analyze evidence in student work to help make more effective instructional decisions. To that point almost every session involves analyzing sets of student work and/or work from participant classrooms (Training Items or OGAP pre-assessments). While you can use the sets of student work in the workshop materials, the materials have greater relevance if the work comes from the participants' classrooms. Importantly, as teachers analyze student work we ask participants to be constantly thinking about three questions.
    a. What do you know from the evidence in student work that can be built upon?
    b. What issues or concerns are evidenced in student work?
    c. What are instructional implications of the evidence in student work?
7. **OGAP and the CCSSM:** All OGAP materials and professional development are aligned with the CCSSM. In particular, there are two sessions targeted at the CCSS as well as other places where relevant CCSSM standards are identified within a workshop**.**
8. **A Focus on Fractions: Bringing Research to the Classroom** (Petit, Laird, & Marsden (2010) is used throughout the workshop. The Facilitator Notes indicate which chapters should be read by facilitators prior to implementing a session. In addition, *if possible*, participants should read Chapter 1 in advance of the workshop. Use your professional judgment on how best to use the other chapters with participants. It is ideal if participants read the chapters as the workshop progresses (See Appendix C), but it has been our experience that if it is a workshop format not all teachers complete the reading so you cannot count on it.
9. **OGAP Professional Development Instructional Strategies:** These professional development materials utilize a range of instructional strategies designed to engage all participants *in thinking about* the important aspects of the workshops. Strategies such as:
    a. **Think, Pair, Share:** "The Think-Pair-Share strategy is designed to differentiate instruction by providing students time and structure for thinking on a given topic, enabling them to formulate individual ideas and share these ideas with a peer. This learning strategy promotes classroom participation by encouraging a high degree of pupil response, rather than using a basic recitation method in which a teacher poses a question and one student offers a response. Additionally, this strategy provides an opportunity for all students to share their thinking with at least one other

student which, in turn, increases their sense of involvement in classroom learning. Think-Pair-Share can also be used as in information assessment tool; as students discuss their ideas, the teacher can circulate and listen to the conversations taking place and respond accordingly." http://www.readwritethink.org/professional-development/strategy-guides/using-think-pair-share-30626.html. The think, pair, share strategy is used throughout OGAP training.

b. **Group work:** We suggest that participants be in groups of not more than 3 or 4 people for the most effective use of group work.

c. **Questioning:** Questioning is used throughout all OGAP sessions as a strategy to deepen understanding of targeted concepts and ideas. In some cases probing questions are provided. However, facilitators should not limit themselves to those questions provided if opportunities arise.

d. **Sharing Solutions:** The point of sharing solutions is to help deepen understanding of a concept. The point is NOT to give participants an opportunity to participate. For this strategy to be effective the facilitator must carefully select solutions to share with the mathematical goal in mind. See Session 1 Facilitator Notes, page 4 of the for an example.

e. **Poster Sessions:** The point of poster sessions is to get *all* participants to think deeply about an idea or concept. We have found that the depth of discussion and thought increases when participants have to commit their ideas to a public poster. In addition, poster sessions are designed to synthesize ideas and concepts. See Session 3 Facilitator Notes, page 9 for one example of how to debrief a poster session.

f. **Problem Solving:** Many of the sessions start with a set of problems or a single problem for the sole purpose of engaging participants in the mathematical ideas related to the session. Examples include: Sessions 1, 4, 6, 9, 10, 11, 12, 13, and 14. Providing participants ample time to engage in the problems, activity, or sets of problems in these sessions is critical.

g. **Formative assessment:** Every session is designed to help the facilitator gather evidence about participant learning to guide their facilitation: a) listening and observing during individual work; b) listening and observing group discussions; c) analysis of posters as they are developed and discussed; d) Full group discussion; and, e) embedding problems into instruction as needed as entry or exit cards.

h. **Using daily workshop evaluations:** Appendix B contains a sample of a daily evaluation. The point of the evaluation is for you to understand what is working and what needs modification as the workshop progresses along with eliciting any questions that participants have that need clarification. We have instituted a daily protocol of opening the following

day's session by reviewing the evaluation information, addressing questions and concerns, and explaining any adjustments that have been made to the workshop as a result of the feedback. We suggest that you make a three slide power point: What's working; Issues/Concerns; Questions.

It is important to be explicit as the workshop progresses about the different instructional strategies you are using; what they are and why they are important. Most importantly, be clear that these same strategies can be used when participants are working with their own students.

SOME SUGGESTED READINGS ABOUT INSTRUCTIONAL STRATEGIES:

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn*. Sausalito, CA: Math Solutions.

Lamberg, T. (2013). Whole Class Discussions: Improving In-depth Mathematical Thinking and Learning. Pearson Publishing.

Smith M., & Stein, M.K. (2011) *5 Practices for Orchestrating Productive Mathematics Discussions*. National Council of the Teachers of Mathematics, Reston, VA.

Walsh, J., & Sattes, B. (2005). Quality Questioning: Research-based practice to engage every learner. Corwin Press, London, England.

Wiliam, D. (2011). *Embedded Formative Assessment.* Solution Tree Press, Bloomington, IN.

Features of OGAP Fraction Professional Development

CONTENT FOCUSED

Sessions 1, 4, 6, 9, 10, 11, 12, 13, 14 begin with participants engaging in solving a problem related to the math topic. The purpose of these problems is to help participants solidify the understanding of the concept under consideration.

CCSSM EMBEDDED IN SOME SESSIONS

While the CCSSM is briefly introduced in Session 0, Sessions 2, 3, 5, 10, and 12 provide a more in-depth understanding on key topics.

[Note: Session 7 is an optional session focused on an in-depth analysis of the CCSSM across grades.]

UTILIZES *A FOCUS ON FRACTIONS: BRINGING RESEARCH TO THE CLASSROOM* (PETIT ET AL, 2016) AS A PART OF THE SESSIONS

- Session 2 – Using Visual Models: Read pages 1 – 8
- Session 3 – Equivalence: Introducing the elements of the book I particular the OGAP fraction Progression.
- Session 4 – Compare and Order Fractions: Uses Study Link 7 – 9.
- Session 5 – Equipartitioning: Reading of the whole chapter.
- Session 6 – Number Lines: Analysis of Figures 6.22, 6.17, 6.10, 6.10, and 6.29 and the Measurement Section.
- Session 10 – Addition and Subtraction: CCSSM and OGAP Fraction Progression sections.
- Session 11 – What is the Whole?: Candy Bar Vignette
- Session 12 – Multiplication: Analysis of Figures 10.1, 10.7, and 10.9 for item design to elicit over generalization and reading on Page 172.
- Session 13 – Division: Analysis of Figure 10.17 (Invert and multiply linked to a contextual problem)

HAS OPTIONS FOR ANALYZING STUDENT WORK

- **OGAP Annotated Sets of work** – Sessions 1, 3, 4, 5, 6, 10, 12, and 13
- **Opportunity to analyze pre-assessments** from participant classrooms – 3, 4, 5, 6, 10, 12, and 13
- **Case Studies focused on ongoing** use of OGAP Session 3, 4, 6, and 10.

INCLUDES OPTIONAL USE OF EXIT CARDS AT END OF SESSIONS

At the end of each session have participants write down 3 – 5 big ideas from the session. Use this to guide your planning.

Session 0: Introduction to OGAP Fractions

This session is designed for those participants who have already completed one other OGAP professional development; Multiplicative Reasoning or Proportionality. If participants have NOT completed another OGAP training than use Session 0 from Multiplicative reasoning and insert Slides 2 - 4 for 14-16 of the Multiplicative Reasoning Session 0.

The focus of this session is to engage participants in a small case that targets the importance of students developing procedural fluency with fractions with understanding in order to engage in a range of middle school topics.

SESSION SEQUENCE

A. Overview of OGAP Fraction Professional Development, **slide 2** (3-5 min)

B. Case Study–The Middle School Dilemma, **slide 3** (20 min)

C. CCSSM and Fractions, **slide 4** (10-15 min)

D. The OGAP Fraction Progression Introduction, **slide 5** (8-10 min)

Session 1: Fractions Are Numbers

This session focuses on a major misconception that permeates fraction understanding – inappropriate whole number reasoning. Students often see fractions as two whole numbers, not a single number representing a value. (Behr, Wachsmuth & Post, 1984; Saxe, Gearhart, & Seltzer, 1999*;* VMP OGAP, 2005). This misunderstanding interferes with students learning new fraction concepts and solving related fraction problems.

The three big ideas of this session are: 1) Fractions are numbers that extend the number system beyond whole numbers; 2) Students often use whole number reasoning inappropriately when solving problems involving fractions; 3) Inappropriate use of whole number reasoning when solving problems involving fractions ***is not inevitable.*** (Petit, Laird, Marsden, & Ebby, 2016)

In the session participants will solve a problem designed to elicit inappropriate whole number reasoning, be introduced to the research about inappropriate whole number reasoning, review samples of student work that exemplify the error, and review one student's pre- and post-assessment. In addition, participants will be introduced to the OGAP Fraction Progression and will review the student work they brought from their classrooms for evidence of inappropriate whole number reasoning.

It is strongly suggested that facilitators read Chapter 2 in *A Focus on Fractions: Bringing Research to the Classroom* (Petit, Laird, Marsden, & Ebby, 2016) before beginning this session.

SESSION 1 SEQUENCE

Part I: Warm-up Problem

A. Solve problem 1/12 + 7/8 is closest to… and debrief, slide 2 (25 min)

Part II: Overview of Research with student work examples

B. Review research and example student work, slides 3-9 (10-15 min)

Part III: Introduction to OGAP Framework with first analysis of student work— Participants will be introduced to the OGAP Fraction Progression and analyze the different levels on the framework.

C. Introduction to OGAP Framework, slide 10 & OGAP Fraction Framework (10 min)

D. Karen's Pre/Post Assessment, **slide 11 & 1B student work** (20-25 min)

E. Your Class, **slide 12** (15 min)

Part IV: OGAP Study and Whole Number Reasoning

F. Research from OGAP study, **slides 13-14** (5-10 min)

Session 2: Using Visual Models

In this session participants will be introduced to mathematics education research related to the importance of using visual models to build fraction concepts. The session will focus on three big ideas: 1) The use of visual models should permeate instruction, not just be an incidental experience, but a way of thinking and solving problems, and developing fraction concepts; 2) Students should interact with a variety of visual models that differ in perceptual features; and, 3) Use of visual models is a means to the mathematics, not the end (Petit, Laird, Marsden, & Ebby, 2015).

Participants will read a section of Chapter 1 in *A Focus on Fractions: Bringing Research to the Classroom* (Petit, Laird, Marsden, & Ebby, 2016), engage in an activity that focuses on the perceptual features of visual models, analyze the CCSSM from a fraction visual model perspective, and analyze their math program materials for use of visual models to develop understanding of fraction concepts and fluency.

SESSION 2 SEQUENCE

Part I: Using visual models is a means to the mathematics, not the end

A. Reading in *A Focus on Fractions* and First to Last, **slide 4 and first to last PowerPoint** (30 min)

Part II: Perceptual features of visual models

B. Analyzing perceptual features of visual models, **slides 5-6** (30 min)

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Part III: Analyzing instructional materials

C. CCSSM and use of visual models, **slide 7** (20-30 min)

D. Instructional Link, **slides 8-9** (30 min)

Part IV: Review of research

E. Review research, **slides 10-11** (5 -10 min)

Session 3: Equivalence

The focus of this session is on developing understanding of equivalent fractions. The big ideas of the session are: 1) Saying that two fractions are equivalent is saying that the two fractions are different names for the same number; 2) There are an infinite number of different names for a given fraction; and, 3) Understanding equivalence and having an efficient strategy to find equivalent fractions are critical as students encounter problems involving comparing and ordering, and operating with fractions.

Participants will create a preliminary definition of equivalent fractions, be introduced to the research regarding the potential consequences of learning procedures without understanding, use quick images and unitizing to build equivalence concepts, and explain the procedure for finding equivalent fractions using visual models.

This session can be done following the visual models session or any time before you begin working with operations. Because of its interactive nature using subitizing and unitizing quick images you may want to do this session right after lunch perhaps using the time before lunch for Parts I–III. Part IV is an intensive session so we suggest NOT doing this session at the end of a day unless you end with Parts I–III and start the next day when participants are fresh with Part IV.

SESSION 3 SEQUENCE

Part I: Definition of equivalent fractions and research on anchoring procedures with understanding

A. Developing a definition of equivalent fractions, **slide 2** (10 min)

B. Introduce research on importance of anchoring procedures on understanding, **slide 3** (3-5 min)

Part II: Quick images – using subitizing and unitizing to understand equivalence

C. Quick Images and subitizing, **slides 4-11** (10 min)

D. Unitizing and equivalence, **slides 12-13** (5-8 min)

Part III: Research on importance of anchoring procedures in understanding

E. Introduce math education research on importance of anchoring procedures to understanding, **slide 14** (3-5 min)

Part IV: Using visual models to understand procedures for equivalent fractions

F. Developing an understanding of procedures for finding equivalent fractions, slides **15-17** (30-40 min)

Part V: Analyzing student work

- **Option A**: Analyze pre-assessment question on, **slide 18** (25 min)
- **Option B**: Complete Equivalence Case Study focusing on using exit questions to inform instruction (60 min)

Session 4: Compare and Order Fractions

The focus of this session is on the different strategies that can be used to compare and order fractions.

In this session participants will compare 9 different sets of fractions designed to elicit a range of reasoning strategies, be introduced to the research about the different classes of fractions and the different strategies students can use to compare and order fractions, use the OGAP sort to analyze a set of student work, analyze the student work from their classrooms* involving comparing and ordering fractions, and analyze their math program for response to research.

*Alternately participants will engage in a case study focused on using ongoing questions to inform instruction.

This session should be completed after the Session 2: Visual Models and before sessions focused on operations. Part V (analyzing instructional materials) can be completed during a PLC, but is important to do before a unit of instruction on comparing and ordering fractions.

SESSION 4 SEQUENCE

Part I: Warm-up – Compare and order using a range of strategies

A. Compare nine pairs of fractions, **slide 2** (30 min)

Part II: Review of relevant research

B. Review of research, **slides 3, 4, & 5** (15-20 min)

Part III: Engineering Questions (Note: If the facilitator is using Option 3 (Case Study) in Part IV, skip this activity. In addition, this activity can be completed as a part of a PLC.)

C. Review OGAP items on back of Framework, **slide 6** (10-15 min)

Part IV: Analyzing evidence in student work—There are multiple options for engaging participants in analysis of student work using the OGAP Fraction Progression. **Option 1** provides a common set of student work for analysis. **Option 2** involves analyzing a pre-assessment question from participants' classroom. **Option 3** is a case study designed to promote understanding of how to use ongoing questions to inform instruction and planning. Options 1 and/or 3

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can be used for training that occurs during the summer when participants do not have student work from their classrooms. Options 1 and/or 3 can also be used in a PLC if the focus during the training is on analyzing participant pre-assessments.

- **Option 1**: Review OGAP Set of work, **slide 7** (20-30 min)
- **Option 2**: Review work from participant classroom–Focus: Pre-assessment, **slide 8** (30 min)
- **Option 3**: Compare and Order Case Study–Focus: Ongoing questions to inform instruction, **see Case Studies** (60 min)

Part V: Analyzing instructional materials

D. Analyzing math program, **slide 9** (20 min)

Session 5: Equipartitioning

This session focuses on equipartitioning. Researchers consider equipartitioning fundamental to developing understanding of fraction concepts and generalizing fractions concepts (Lamon, 1999; Behr & Post, 1992; Comfrey et al, 2011). Equipartitioning is needed when identifying fair shares, identifying fractional parts of an object or set of objects, comparing and ordering fractions, locating fractions on a number line, understanding the density of rational numbers, evaluating equivalence of two fractions, operating with fractions and measurement (Lamon, 1999).

In this session participants will read sections of Chapter 4: Equipartitioning (Petit, Laird, Marsden, & Ebby 2016), identify important ideas and instructional implications. Participants will be assigned sections of Chapter and then using the Jigsaw strategies share the big ideas of the section they read with their table group.

SESSION 5 SEQUENCE

Part I: Eliciting and sharing "big ideas"

A. Reading assignments, **slide 2** (15-20 min)

B. Debriefing "What is equipartitioning and why is it important?" **slide 3** (5-10 min)

C. JIGSAW, **slide 4** (20-30 min)

Part II: Analyzing instructional materials (Optional)–Can also be completed in PLC

D. Partitioning and math programs, **slide 7** (15 min)

Session 6: Number Lines

This session focuses on the use of number lines to build fraction concepts. Research suggests that the number line may be fundamental in helping students understand a fraction as number, understand unit fractions, develop concepts of equivalence,

magnitude, density of rational numbers, and operations (Behr & Post, 1992; Saxe et al, 2007; Petit et al, 2016).

In this session participants will solve number line problems and identify difficulties students might encounter when solving number line problems. In addition, participants will be introduced to the math education research about the challenges students have using number lines, review common errors that students make using number lines, analyze student work, explore the relationship between number lines and measurement concepts, and analyze instructional materials for use of the number line. The last two parts of this session are optional and can be completed as a part of PLCs during the ongoing support of OGAP. Also note that there are 3 options for analyzing student work.

Session 6: Number lines can be completed at any time after the Sessions 1, 2, and 4. Given the importance of students thinking about fractions as quantities the number line session is critically important for all teachers. The session involves 5 distinct parts. If time in a session does not allow for completing all 5 parts of the activity in one time period, it is suggested that Parts I – III should be done together. Parts IV and V can be done at another time during PLCs or other shorter periods of time.

SESSION 6 SEQUENCE

Part I: Number Line Warm-up

A. Number Line Warm-up, **slide 2 & 6A handout** (20-25 min)

Part II: Number line research and use in classrooms

B. Number line use in the classroom, **slide 3** (15 min)

C. Number line research, **slides 4-13** (20 min)

Part III: Analyzing evidence in student work—There are multiple options for engaging participants in analysis of student work using the OGAP Fraction Progression. **Option 1** provides a common set of student work for analysis. **Option 2** involves analyzing a pre-assessment question from participants' classroom. **Option 3** is designed to promote understanding of how to use ongoing questions to inform instruction and planning. Options 1 and 3 can be used for training that occurs during the summer when participants do not have student work from their classrooms. Options 1 and 3 can also be used in a PLC if the focus during the training is on analyzing participant pre-assessments.

D **Option 1**: Review OGAP Set of work, **slide 7** (20 min)

Option 2: Review work from participant classroom–Focus: Pre-assessment, **slide 8** (30 min)

Option 3: Number Line Case Study–Focus: Ongoing questions to inform instruction, **see Case Studies** (60 min)

Part IV: Analyzing instructional material

E Reviewing the math program, **slide 16** (10 min)

Part V: Relationship between number line and measurement

F Relationship between the number line and measurement, **slide 15** (20 min), optional

Session 7: OGAP and the CCSSM

This session is OPTIONAL. It should be used if the CCSSM sessions embedded in other OGAP sessions have not been used. In addition, look ahead to Session 8 that refocuses on the CCSSM from a middle school perspective. If you are working with only middle school teachers (grades 5-8) then we suggest using session 8 instead of Session 7. If you are working with only elementary teachers than we suggest using Session 7 and a review of Slide 4 in Session 8.

This session is focused on the CCSSM and fractions. In the session the focus will be on three questions:

1) How do fraction concepts and fluency develop across grades 1- 6?
2) How are visual models used as concepts develop?
3) What content at the middle school assumes proficiency with fractions?

Participants will analyze a grade level of the CCSSM in the Operations and Number Fraction strand, write a general description of what is expected at that grade level, and provide some problems that exemplify the description for grades 1-6. For grades 6 – 8 participants will identify content that is dependent upon on strong fractional knowledge and fluency, and provide some examples that show why there is a fraction demand. Participants will make posters by grades and then display across the grades to help answer the questions.

This session involves two distinct parts: a small group analysis by grade and a cross grade full group analysis. The session can be completed at any time after Sessions 1 and 2. This session involves a lot of detail so it is suggested that you do NOT do the session at the end of a day.

SESSION 7 SEQUENCE

Part I: Understanding and exemplifying the fraction stand of the CCSSMM–by grades

A. Session Overview, **slides 2-4** (10 min)

B. Work Session 7B (40 min)

Part II: Debriefing–across grades

C. Debriefing Estimated, **slide 5** (15 min)

Session 8: Fraction operations overview: OPTIONAL Session

This session is designed for a workshop that focuses JUST on operations with fractions without completing all the foundational sessions.

The purpose of this session is to introduce participants to the major research that underpins the development of fraction operations and the importance of students entering middle school with understanding of fraction concepts and procedural fluency. Participants will: review goals for the two day workshop focused on fractions operations; review the major research impacting operations with fraction operations; consider the importance of procedural fluency with fractions by middle school; and, establish a working definition of procedural fluency.

SESSION 8 SEQUENCE

Part I: Goals, overview, the CCSS and research considerations

A. The Middle School Fraction Dilemma, **slides 2-5** (20-25 min)

B. Major Research Considerations, **slide 6** (3 min)

Part II: What is procedural fluency?

C. What is procedural fluency? **slides 7-9** (10-15 min)

Part III: Workshop Foci

D. Workshop Foci, **slide 10** (3 min)

Session 9: Decimal Maze

The Decimal Maze is a game-based activity designed to engage participants in a major over generalization that students (and adults) make regarding multiplication and division. Researchers indicate that *students believe that multiplication of two numbers results in a larger number and division results in a smaller number* (Bell, Fischbein, & Greer, as cited in Harel, Behr, Post, & Lesh, 1994) Participants will play the game and then develop statements (conjectures) about the impact of multiplication and division of whole numbers with decimal fractions and fractions. [Note: This activity has been adapted from "Too Big or Too Small" **http://illuminations.nctm.org/lessons/6-8/numbersense/BigSmall-AS-Maze.pdf**. Permission is not needed to replicate the Decimal Maze.]

The Decimal Maze has two distinct parts: playing the game and understanding the impact of fractions operations; and, using visual models to understand the impact of division. This session should not be broken up in different time periods if possible.

SESSION 9 SEQUENCE

Part I: Playing the game and debriefing

A. Play the Decimal Maze to obtain the largest value, **slide 2** (10-15 min)

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B. Debrief the highest value, **slides 3** (5-10 min)

Part II: Using Models

C. Use visual models to understand the impact of multiplication and division, **slides 4-7** (20 min)

D. Bringing it together and extending understanding, **slides 8-9** (10-15 min)

- **Option 1**: Refining a conjecture
- **Option 2**: Impact of different operations

Session 10: Addition and Subtraction of Fractions

All the operation sessions focus on two pieces of very important math education research about developing procedural fluency:

1) Premature experience with formal procedures may lead to symbolic knowledge that is not based upon understanding, or connected to the real world. This may impede students' number and operation sense (Kiernan, as cited in Huinker, 2002).
2) Researchers express concern that this type of learning can be "highly dependent on memory and subject to deterioration." (Kieran, 1988) This deterioration results when symbol manipulation is emphasized to the relative exclusion of conceptual understanding and adaptive reasoning. (NRC, 2001)

The session provides examples of strategies that researches indicate will provide a strong foundation for fraction understanding: a) Begin with simple contexts; b) use estimation and other reasoning strategies to develop understanding, and, c) explore and build understanding with visual models, understanding of properties, and unit fraction understanding.

In this session participants will use estimation to mentally solve some addition and subtraction problems, analyze some word problems with simple contexts, explore relationships between models, unit fractions, and common denominator strategies to understand the CCSSM for addition and subtraction of fractions, analyze evidence in training items, and review their math program.

This session can be considered as 4 distinct parts as described below. It is best done as one 2.5 hours session, but the analysis of instructional materials and CCSSM can be completed in a PLC setting. The order is important in this workshop.

SESSION 10 SEQUENCE

Part I: Laying a foundation

A. More or less than 1? Warm-up, **slide 2** (10-15 min)

B. Research introduction, **slides 3-4** (3 min)

C. Begin with simple contexts, **slide 5** (10 min)

D. Use reasoning about magnitude, **slides 6-7** (10 min)

Part II: Understanding addition and subtraction and its link to the CCSSM

E. Explore operations using visual models, unit fraction reasoning, and common denominators, **slides 9-11** (30 min)

F. Properties of operations, **slides 12-13** (15-20 min)

G. Understanding CCSSM standards for addition and subtraction of fractions, **slides 14-15** (15-20 min)

Part III: Analyzing student work

H. **Option 1**: Analyze pre-assessment/training items, **slides 16-17** (30 min)

Option 2: Addition and Subtraction Case Study (60 min)

Part IV: Analyzing instructional materials

I. Analyze math program, **slide 18** (15-20 min)

Session 11: What is the whole?

This session focuses on two big ideas: 1) When comparing fractions it is important to assure that the solution considers the size of the wholes being compared; and, 2) Using visual models to represent a problem situation in which the wholes are not the same size.

SESSION 11 SEQUENCE

A. Solving the Westport–Danville problem, **slide 2** (30 min)

B. Debriefing the Westport–Danville problem, **slide 3** (20 min)

C. The Candy Bar Vignette, **slide 4** (5-10 min)

Session 12: Multiplication of Fractions

This session is designed to help teachers learn how to bring meaning to multiplication of fractions. It builds on the knowledge about the impact of multiplication and division on magnitude of results developed in the Decimal Maze activity and the Danville –Westport problem. Researchers indicate that multiplication and division of fractions are among the most complicated fraction concepts that elementary students encounter (Fendel, as cited in Tirosh, 2000). For this reason researchers strongly suggest that students should experience a range of multiplication and division situations and build fluency with understanding through use of visual models and estimation (Kieren, as cited in Huinker, 2002; Aksu, 1997). To this end the session involves participants using visual models to represent fraction multiplication problems that leads to understanding of the traditional

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algorithms for multiplication. Finally, participants analyze evidence in student work using the OGAP Fraction Progression.

IMPORTANT: See Session Sequence (page 2) for suggestions on using the materials in this session for different grade levels.

This workshop can be divided and implemented in 3 distinct parts. Depending upon the time available it can be done as one 2 +/- session hour session or in the chunks as smaller workshops as described below. The order of the Parts is important. Parts I and II are best done in the same session. Analyzing the instructional materials can be completed in a PLC or other small chunk of time as a unit of multiplication of fraction begins. All aspects of this session are appropriate for grades 5 and higher. Part I A – C are targeted for multiplication CCSSM requirements at grade 4. Therefore, if there are time constraints or you are working with just a grades 3 – 4 teacher group, we suggest using Part I A – C up to slide 11.

SESSION 12 SEQUENCE

Part I: Introduction and using visual models and multiplication

A. Introduction and fraction maze, **slides 2-3** (20 min)

B. Shade 5/8 of grid, **slides 5-7** (20-30 min)

C. Multiplication of a whole number by a fraction, **slides 6-11** (20-30 min)

D. Using visual models to understand multiplication of fractions, **slides 12-19** (40 min)

Part II: Analyzing student work (Optional)

E. Analyzing student work, **slide 20** (30 min)

- **Option 1**: Student work from participant classroom
- **Option 2**: OGAP set of student work

Part III: Analyzing Instructional Materials

F. Analyzing instructional materials, **slide 21** (15 min)

Session 13: Division of Fractions

This session is designed to help teachers learn how to bring meaning to the division of fractions and builds on understanding about the impact of multiplication and division on magnitude of results. Researchers indicate that multiplication and division of fractions are among the most complicated fraction concepts that elementary students encounter (*Fendel, as cited in Tirosh, 2000*). For those reasons researchers strongly suggest that students should experience a range of division situations and build fluency with understanding through use of visual models and estimation (Kieren, as cited in Huinker, 2002; Aksu, 1997). To this end the session involves participants using visual models to represent division problems as well as activities that develop understanding of the

different meanings of division (partitive and quotative) and remainders. Next participants analyze evidence in student work using the OGAP Framework and engage in discussions about instructional next steps based on the evidence. Finally, participants analyze patterns and relationships in sets of division problems and in engaging in a context problem designed to lead toward a mathematical understanding why "invert and multiply" and the "common denominator" algorithms work for solving fraction division problems.

This workshop can be divided and implemented in 4 distinct parts. Depending upon the time available it can be done as one 3 hour session or in the chunks as smaller workshops as described below. The order of the Parts is important. In particular, Part I sets the foundation about the types of division and work with remainders. Pat I should be completed before other parts of this workshop.

SESSION 13 SEQUENCE

Part I: Understanding partitive and quotative problems and interpreting remainders

A. Solving division word problems, **slides 2** (20-30 min)

B. Debriefing word problems and solutions poster session, **posters on wall & slide 3** (20-30 min)

C. Division interpretation of fractions, **slides 4-7** (20-30 min)

Part II: Analyzing student work

D. Intro to analyzing student work, **slide 8** (10 min)

E. **Option 1:** OGAP set of work, slides 9-10 & student work 6A-B (20-25 min)

F. **Option 2:** Classroom set of student work**, slides 9-10** (30-40 min)

Part III: Developing understanding of division algorithms

G. Developing understanding of division of fraction algorithm– Engineering a discussion, **slides 11-16** (30-40 min)

Part IV: Analysis of instructional materials

H. Analyzing Instructional Materials, **slide 17** (20 min)

Session 14: Density of Fractions

This session is focused on the density of fractions. That is, there are an infinite number of fractions between any two numbers. In the session participants will be asked to use a range of strategies to find three fractions between 4 different sets of fractions they are given. They then make a group poster and as a group analyze the different strategies used. Finally, participants review research about student lack of understanding about density, and then analyze evidence in student work.

The session should be completed as 1 session of about 1.5 hours. It does NOT have be done as a part of the larger session, but should be completed after the other operation sessions are completed.

SESSION 14 SEQUENCE

A. Solve problems and create a group poster, **slide 2** (20 min)
B. Debrief poster solutions, **slide 3** (25 min)
C. Definition and Research, **slides 4-5** (5 min)
D. Student solutions, **slides 6-10** (15 min)

Session 15: Navigating the Item bank – in development

In this session participants will become familiar with how the item bank is organized. Participants will form small groups and examine one section of the item bank to help fellow participants understand it more thoroughly.

In this session participants will become familiar with how the item bank is organized. Participants will form small groups and examine one section of the item bank to help fellow participants understand it more thoroughly.

SESSION 15 SEQUENCE

A. OGAP Item Bank Review**, slides 2–10** (40-50 min)
B. Case Studies, **slide 11** (30 min), optional

Appendix A: Implementation Ideas at a Glance

Session Number	When in Sequence of workshop	Teachers of grades*	Special Notes
1: Fractions Are Numbers	After an introduction to OGAP and Learning Progressions	2-6	Complete all aspects of this session before moving to next session.
2: Use of Visual Models	After Session 1	2-6	Complete all aspects of this session before moving onto the next session.
3: Equivalence	Any time after Sessions 1 and 2 and before Session 9 Addition and Subtraction	3-6	Parts I – III are ideal for a half hour before lunch or at the end of a day. Do NOT do Part IV at the end of the day.
4: Compare and Order	Any time after Sessions 1 and 2 and before Session 6 – The number line	3-6	Complete all parts except Part IV – analyzing instructional materials can be completed during PLC time
5: Equipartitioning	Any time after Sessions 1 and 2	2-5	There are 2 ways to implement this session depending upon whether participants have read Chapter 4 (Petit, Laird, and Marsden, 2010) See Facilitator Notes for this session.
6: Number Line	Any time after Sessions 1, 3, and 4	3-6	Parts IV and V can be completed during PLC time.
7 and 8: CCSS and Fractions	Any time	All	OPTIONAL: Choose between Sessions 7 or 8. If you use Session 7 also complete Parts II and III of Session 8 on procedural fluency
9: The Decimal Maze	First session of the operations workshop	3/4-6	Do all part so this session before moving onto the rest of the operation sessions
10: Addition and subtraction	After Session 1-6 and 9 is completed and before Session 12 and 13	3/4-6	Part IV – analyzing instructional materials can be completed during PLC time
11: What's the Whole?	Any time after Sessions 1 and 2 and before Session 12: multiplication	3/4-6	This is a good warm-up problem that requires participants to model a situation involving different size whole.
12: Multiplication of fractions	After Sessions 9 and 11	3/4-6	Part IV – analyzing instructional materials can be completed during PLC time
13: Division of Fractions	After Sessions 9, 11, and 12	3/4-6	Part IV - – analyzing instructional materials can be completed during PLC time
14: Density of fractions	After Sessions 9, 11, 12,and 13	3/4-6	Can be done in PLC
15: Navigating the item bank	Any time after the Foundational Sessions		Can be completed in PLC or during the workshop

Appendix B: Daily Evaluation

Daily Feedback Sheet

Date:

Which best describes you?

- ☐ Classroom Teacher
- ☐ School or District Mathematics Teacher Leader
- ☐ Other _____

What activities and/or concepts in today's workshop were especially useful for you?

Do you have any issues or concerns about the workshop? If yes, describe.

What questions do you have for the instructors?

Appendix C: Recommendations for use of *A Focus on Fractions: Bring Research to the Classroom* (Petit, Laird, Marsden, & Ebby (2016) if incorporating the book as a part of a course.

Chapter	Recommendations
Chapter 1: Use of Modeling	Read prior to Session 2
Chapter 2: Fractions Are Numbers	Read after Session 1 – select questions from *Looking Back* for reinforcement of ideas
Chapter 3: What is the whole?	Read after Session 2 or 11 - – select questions from *Looking Back* for reinforcement of ideas
Chapter 4: Equipartitioning	Read prior to Session 5 or as a part of Session 5 per Facilitator Notes - – select questions from *Looking Back* for reinforcement of ideas
Chapter 5: Comparing and ordering fractions	Read after Session 4 as reinforcement - – select questions from *Looking Back* for reinforcement of ideas
Chapter 6: Number lines and fractions	Read after Session 6 - – select questions from *Looking Back* for reinforcement of ideas
Chapter 7: The density of fractions	Read after Session 14 – select questions from *Looking Back* for reinforcement of ideas
Chapter 8: Equivalent fraction and comparisons	Read after Session 3– select questions from *Looking Back* for reinforcement of ideas
Chapter 9: Addition and subtraction	Read after Session 10 – select questions from *Looking Back* for reinforcement of ideas
Chapter 10: Multiplication and division	Read after Sessions 11, 12, and 13 - – select questions from *Looking Back* for reinforcement of ideas
Chapter 11: The OGAP Fraction Framework	After Session 1 and 2 **www.routledge.com/books/details/9780415801515/**

FREE DOWNLOADS

Answer Key for Looking Back and OGAP Fraction Framework - in eResources:
https://www.routledge.com/products/9781138816442