

Using Design-Based Research to Develop Vermont's Ongoing Assessment Project (OGAP)

NCTM Research Pre-Session 2008

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Plus about 300 Vermont and Alabama teachers and teachers and about 6000 students who participated in OGAP Exploratory Studies and 2006-2008 scale-up

Active OGAP National Advisory Board

- **Mary Lindquist**, Callaway Professor of Mathematics Education, Emeritus; Past President of the National Council of Teachers of Mathematics
- **Ed Silver**, University of Michigan
- **Judith Zawojewski**, Illinois Institute of Technology

Goals of Session

- Provide an overview and background of OGAP materials and processes
- Illustrate some ways that Design Based Research was used in the development of OGAP (the big ideas, not the details)

Some OGAP Background



The Big Problem – 2003 – Classroom Observations and Interviews Showed that (VMP 2003)

- Teachers rarely monitored students' understanding - prior to or during instruction.
- Teachers believed that students had adequate prior knowledge for the lesson - and that if they did not, it was mostly due to low ability - innate deficiencies.
- Teachers believed that students in the class were learning what the teacher was intending to teach – usually based on the responses of a few students.
- Teachers were often surprised and frustrated when students did poorly on subsequent assessments.
- Teachers attempted to use large scale assessment information to inform instruction and were quickly frustrated

The Charge

- To provide teachers with tools and strategies to monitor student learning as students were learning, not later.

Principles upon which OGAP was Designed

Principle # 1: Build on pre-existing knowledge (How People Learn (2000) National Research Council)

Principle # 2: Learn (and assess) for Understanding (Adding it Up! (2001) National Research Council)

Principle # 3: Use Frequent Formative Assessment (Inside the Black Box, (2001) Black, P, and Wiliam, D.)

Principle # 4: Build Assessment on Cognitive Research (Knowing What Students Know (2001) National Research Council)

OGAP is an intentional and systematic cognitively based formative assessment in mathematics involving:

- Gathering information about pre-existing knowledge through the use of a **pre-assessment**;
- **Analysis of pre-assessment** to guide unit planning; and
- **A continuous and intentional system** of instructing, probing with instructionally embedded questions, analysis, and instructional modification.

Grades 2 - 8

● **Fractions**

● **Multiplicative reasoning**

● **Proportionality**

Supported by...

- Cognitively sensitive pre-assessments;
- Item banks with hundred's of questions;
- Strategies and tools for gathering information about student learning and for making instructional decisions;
- Materials to communicate research; and
- Professional development models.

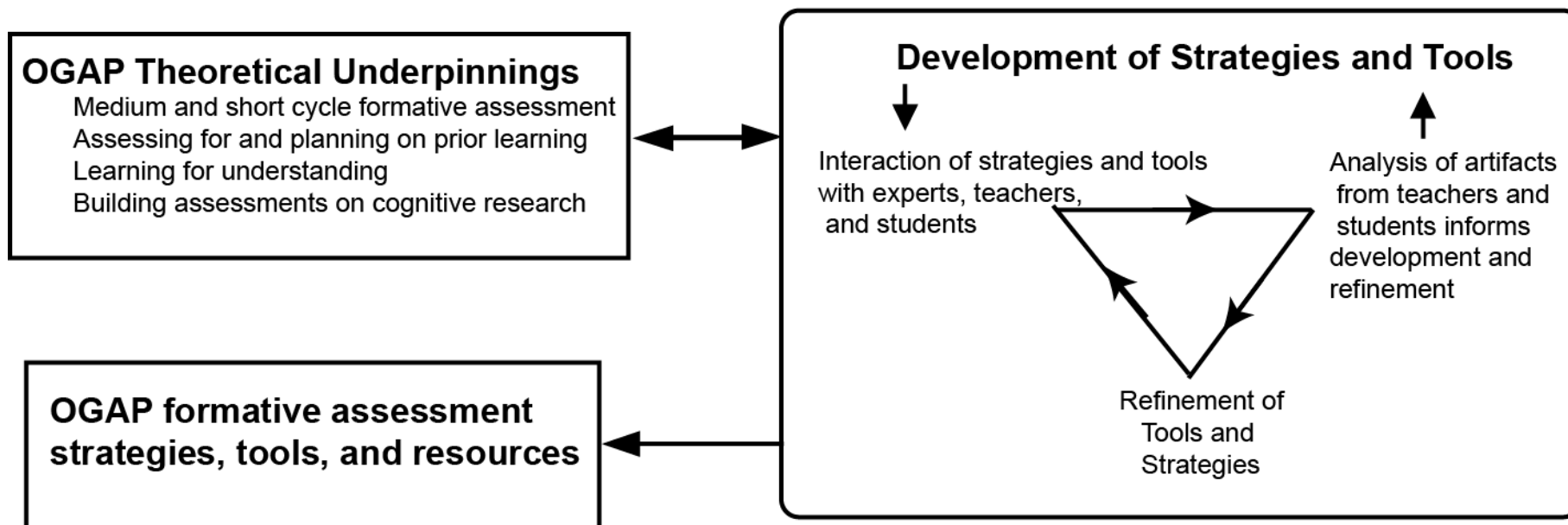
Design Based Research and OGAP

**...by “designing, studying,
and refining a theory based
intervention (OGAP) in the
context of real classroom
settings and contributing
to .”**

(Hake, 2004; Cobb, 2001; Collins, 1992 cited in Designed
Based Research Collaborative, 2003; Schoenfeld, 2007; RAND Mathematics
Study Panel, 2003)



OGAP Design Based Research Model



Framework for OGAP Research and Development

Goal: Help teachers understand

Intervention: Develop tools and strategies linked to goal...

Test: Early cognitive labs...

Revision:

Test: 2004 study...

Revision:

Test: 2005 study...

Revision:

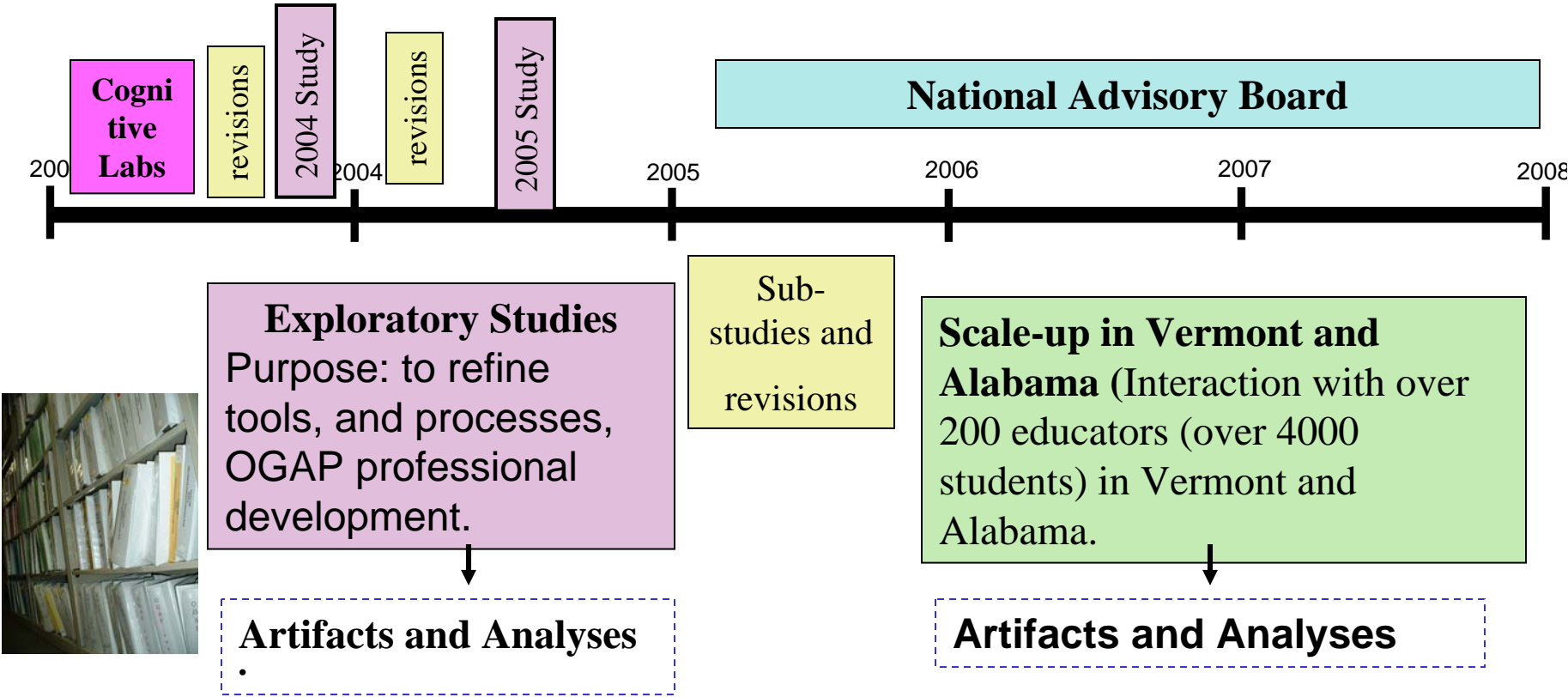
Test: 2006 – 2007 OGAP scale-up...

Revision:

Test: 2008 + ...

Design Committee – school based leaders and teachers, assessment expert, a mathematician (distillation of hundreds of research articles used as the foundation of OGAP tools and resrouces0)

Distillation and Subsequent Instantiation of Research



Design Based Research was used to Inform Development of All Aspects of OGAP

- Tools, strategies, and resources
- Teacher professional development substance and models
- All related supports

An Example – Major Change

From Research used to primarily develop items to a major underpinning of all aspects of the project that ultimately influenced...

- a) Teacher understanding of the evidence in student work;
- b) Teachers understanding of purposes of activities in math program;
- c) Instructional decisions;
- d) First wave instruction

How we communicate research to teachers changed

- From – organized lists of research findings (10 pages)
- To –
 - a) engaging essays/chapters and activities that used student work and case studies to illuminate the research;
 - b) Frameworks that teachers use to sort student work, understand structures of problems, understand their mathematics programs

F2B₁ 2: The number parts in a whole is a factor of the denominator (including an area model that has not been partitioned);

F2B₁ 3: The number of parts in a whole is a multiple of the denominator.

F2C₁: Students move through “Levels of Partitioning” (Pothier and Sawada, 1990, cited in Bezuk and Bieck, 1993, pp.124 – 125)

F2C₁1: Sharing: Students can draw lines down the middle of a region to represent halves (halving).

F2C₁2: Algorithmic Halving: Students draw lines to continue the halving process to obtain fourths, eighths, sixteenths.

F2C₁3: Evenness: According to research it is easier for students to partition models into even numbers that are powers of two, than odd numbers or even numbers that have odd number factors.

F2C₁4: Oddness: Because the halving strategy does not work with odd numbers or even numbers that have odd number factors (e.g., 6, 10), it is more difficult for students to partition models into odd numbers, than even numbers.

F2C₁5: Composition: As students become more flexible in their partitioning and understanding of multiplicative reasoning, they use multiplicative strategies to partition. (e.g., to obtain 12^{th} the student divides fourths into thirds).

F2D₁: Different strategies are used when students are finding fractional parts of the whole where the whole contains multiples of the denominators. (VMP Observation MQ1 – December, 2003)

F2D₁1: Counting/numeric strategy: When finding $\frac{1}{4}$ of a shape that contains 12 parts the students:

F2D₁1a: counts out 12 and then shades 3, or

F2D₁1b: shades every fourth part.

F2D₁2: Visual-geometric strategy:

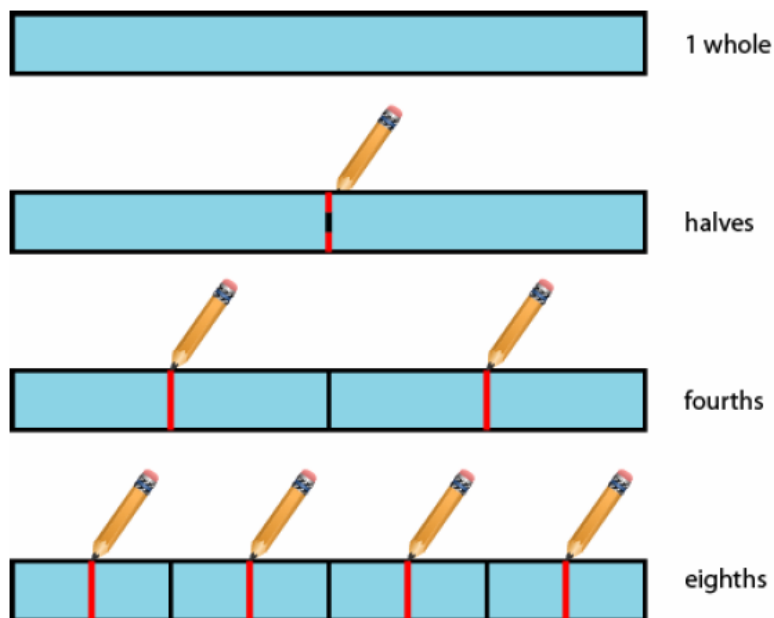
F2D₁2a: finds $\frac{1}{4}$ of the whole shape disregarding the number of smaller divisions within the shape, or

F2D₁2b: shades $\frac{1}{4}$ of each part seeing each part as a whole.

F2E₁: Students have a difficult time determining the whole when they are given just a part. (e.g., 6 is $\frac{1}{4}$ of the whole. What is the whole?). (Behr and Post, 1992)

Algorithmic Halving

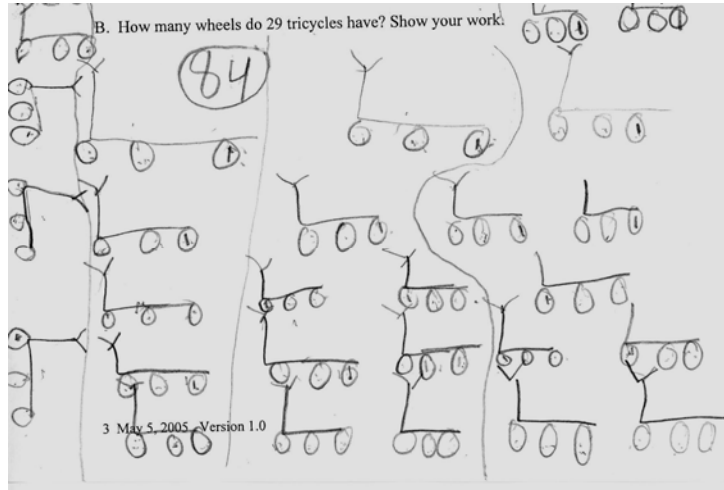
Students usually move easily from *sharing* to *algorithmic halving* which is the process of *continuing the halving process to obtain fourths, eighths, sixteenths, etc.* (Pothier, Y.M., and Sawada, D., 1990, cited in Bezuk and Bieck, (1993)). Fraction strips are used below as examples of the impact of algorithmic halving. Each fractional piece, starting with the whole strip, is *halved* to create the next smaller piece.



Partitioning regions, sets and lines into equal parts that are powers of two (i.e., fractions with denominators of 2, 4, 8, 16, 32...) is easier than partitioning that involves odd numbers or even numbers that have odd number factors. (Pothier, Y.M., and Sawada, D., 1990, cited in Bezuk and Bieck, (1993) This research suggests that students should be introduced to partitioning with fractions whose denominators are powers of two ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$).

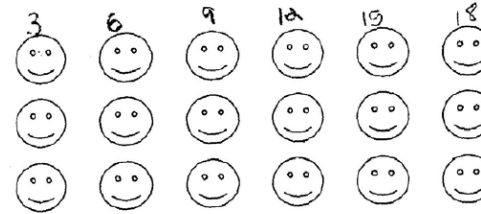
One tricycle has three wheels.

How many wheels do 29 tricycles have?



**Transitional
Multiplicative
Strategy**

Write an equation to match this picture.



$3 \times 6 = 18$ 3, 6, 9, 12, 15, 18

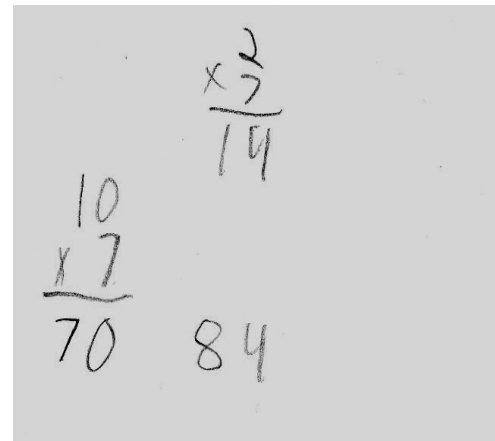
Additive Strategy

Multiplicative Strategy

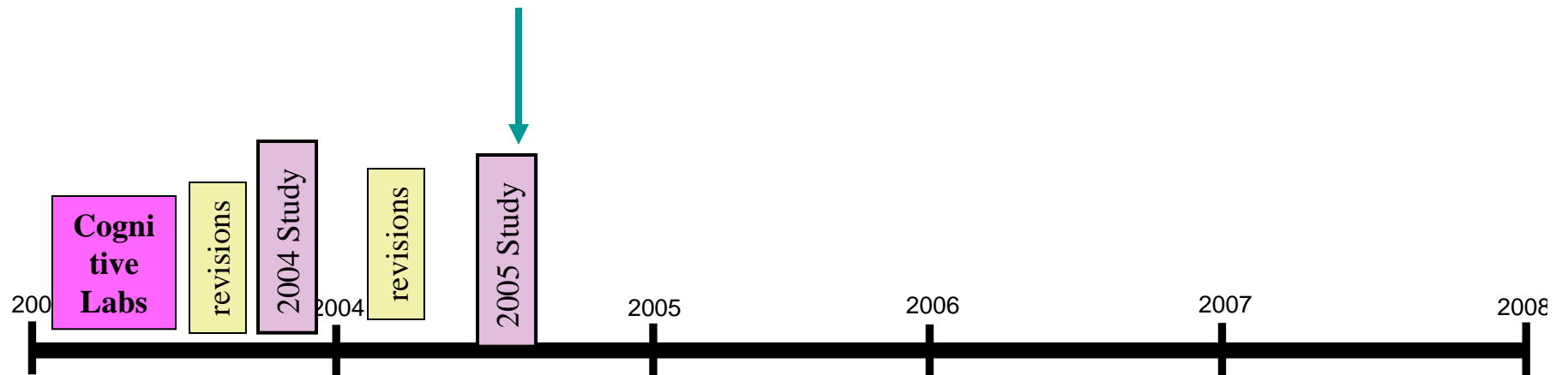
Farmer Brown donated 7 dozen eggs to the senior center.

How many eggs did he donate?

Student
work sort



Tell Modeling Story



Exploratory Studies
Purpose: to refine tools, and processes, OGAP professional development.

Artifacts and Analyses
.

Goal - Help teachers understand how to use models to solve problems and build mathematical ideas.

Intervention: Develop tools and strategies to help teachers understand how to use models to solve problems and build mathematical ideas.

- 1) Developed a bank of items that reflect area, set, and linear models.
- 2) Provided professional development for teachers in understanding the different models and perceptual features of models.
- 3) Provided professional development encouraging teachers to use models to solve problems.

Learning is facilitated when students interact with multiple models (and contexts) that differ in “perceptual” features causing students to continuously rethink the concept (and not to over generalize based upon one model). (Behr, Post and Lesh, 1981 cited in Bezuk and Bieck, 1993; and VMP 2004 Study)

Test: 2005 study...

- 9 Vermont Schools
- Grades 2 – 6
- 63 teachers/classrooms
- Over 1200 students
- 3 student teachers
- Mentors = 10

Used in:

- Intervention centers
- Classrooms
- As a part of Action Research for VMI students

Exploratory Study 2005

Purpose of study: to refine tools, and processes, OGAP professional development.

Participants...

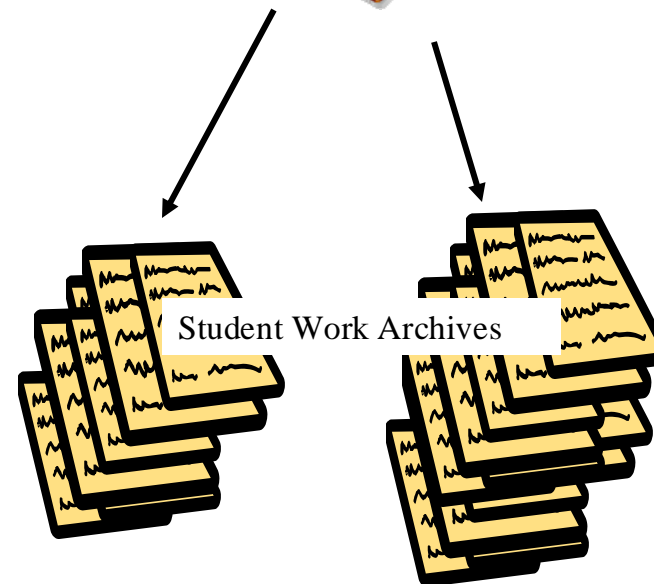
- 6 hours of professional development
- item bank of cognitively sensitive pre-assessments and items
- Met with a mentor once a week (OGAP Committee Member)

Artifacts and Sources of Evidence

- pre-assessments to their students
- Maintained a log
- Maintained a student work archive for every student in their class
- Completed a background survey
- Completed a post survey

(Some) Artifacts to Inform Intervention

- Teacher logs
- Student work archives



Test: 2005 study...

Exploratory Study 2005 - Analysis

Log Sampling by Grade Level: 2005 OGAP Exploratory Study			
	Total Number of Teacher Logs	Randomly Selected Teacher Logs	Percent Sampled
Grade 2	15	8	53%
Grade 3	17	9	53%
Grade 4	15	8	53%
Grade 5	10	8	80%
Grade 6	3	2	66%
Totals	60	35	58%

Student work sample: $n = 1565$ pieces of student work

- 3 students per teacher randomly selected
- 2 questions randomly selected pre and post per student
- All questions embedded in instruction of sampled logs

Sample Teacher Log

Sample of Evidence

Link to OGAP questions

Pre-Assessment Results

Based on the evidence in the student work of the pre-assessment:

- Students need further work on set model - most of my students had no idea how to solve these problems
- Area Model
 - *Guessing as they have no knowledge to form explanation
 - *Unable to divide two objects equally
 - *Knew to shade parts but couldn't divide area equally - they don't understand equal parts (9, 26, 11)
 - *Some success was found dividing into fourths - others were more difficult
 - *Writing notation was incredibly difficult
 - *Unclear whether they understand adjacent parts.

Q4-20, 22, 2

After analyzing this pre-assessment it was very clear that my students need instruction in fractions (both set and area models). Both models seemed equally difficult for my students. We have not reached any parts in the MathLand curriculum that teach fractions, so, I was not terribly surprised by their lack of understanding of fractions.

Since I have given the assessment, I have begun a daily fraction "tune-up." Every day as I begin class, I write a fraction question on the board. We take about 5 minutes daily to discuss the question and sometimes even extend it. We are working on area and set model, and I try to include skills such as writing fractional notation, sharing with odds and evenness, different shaped areas, etc.

These mini-lessons seem to be making a huge difference for my students. There is only one week in the MathLand curriculum that focuses on fractions. The week prior to the fraction lessons is about "sharing-exploring the concept of division." Last year I chose to teach the fractions unit first, and then I went back to the "sharing" unit. I felt that this would be helpful as it would allow us to look more closely at the connections between "sharing" and fractions after my students had a better base knowledge. I plan to begin my unit the same way, again.

	Questions	Student IDs
	4	20, 22, 2
Random Selections	5	26, 10, 11 ↑ no work

(PA1) Analysis
Area Model
Set model
Fourths
Notation
Adjacent parts
Equal sized parts

(PA2) UNIT
Set + area model

(PA3) Instruction
Daily fractions
Area
Set
Notation
Odds and evenness
Models
Different shaped areas

(PA4) Unit
Connection between "sharing" and fractions

Major Categories

Evidence specifics

Agreements between Expert Committee and Study Teachers Strong – What teachers said in logs and what student did on pre and post questions!

	Agreement to Analysis	Agreement to Coding (I, P, S)
Number of Responses Sampled	1320	1357
Total Sampled	1565	1565
Percent Agreement between Reviewer and Study Teacher	84%	87%

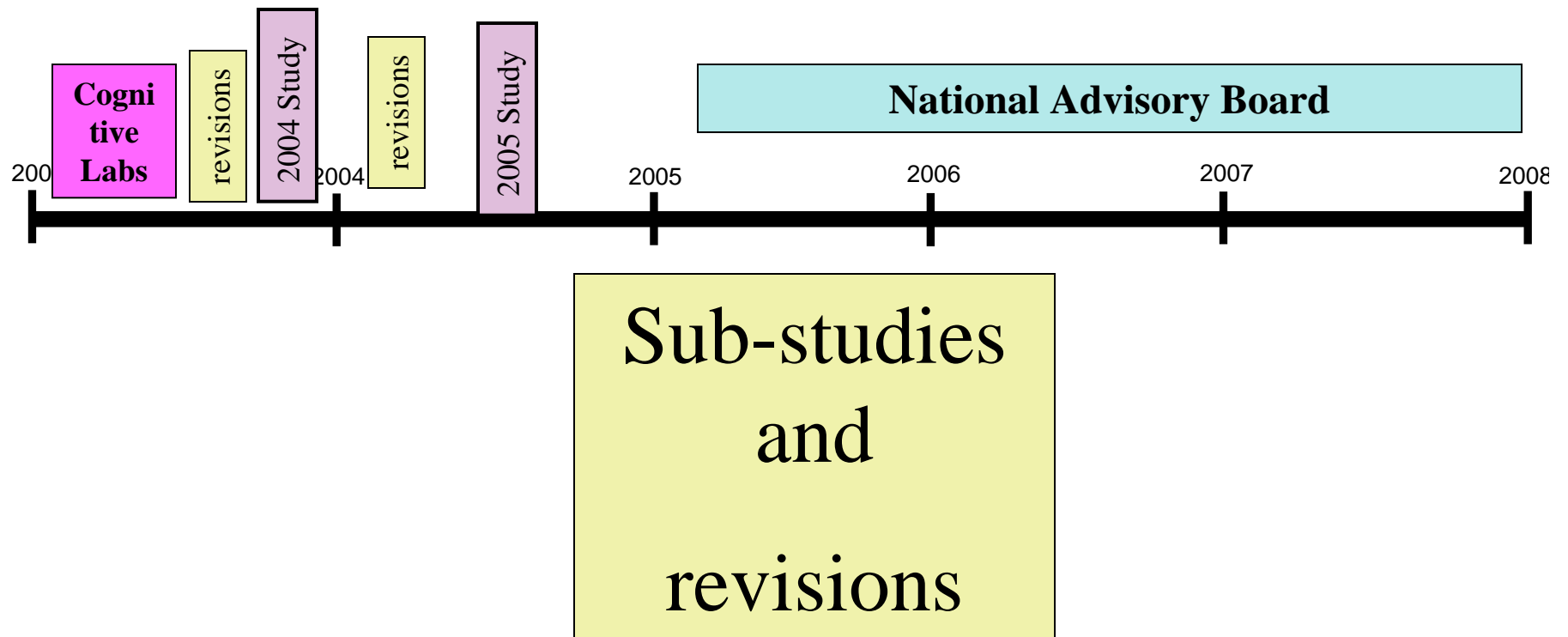
As committee members completed the summer 2005 analysis they made a hypothesis based on observations...

Findings

- 1) The dominant error in pre-assessments appears to be inappropriate whole number reasoning;
- 2) The dominant strategy in correct responses in the post assessment was the use of models.



Design Committee – school based leaders and teachers, assessment expert, a mathematician
(distillation of hundreds of research articles used as the foundation of OGAP tools and resrouces0)



Sample of Evidence

Inappropriate whole number reasoning

According to research, some students may see a fraction as two whole numbers (e.g., $\frac{3}{4}$ as a 3 and 4) inappropriately using whole number reasoning, not reasoning with a fraction as a single quantity.

(Behr, M., Post, T., Lesh, R., and Silver, E. (1983); Behr, Wachsmuth and Post, (1984); VMP OGAP Study (2005))

$\frac{1}{3}$ of the students in Joe's class walk to school.

$\frac{3}{4}$ of the students in Joe's class ride the bus.

Do more students walk to school or ride the bus?

Explain your answer using words and diagrams.

No because 1 person rides the bus and 1 person walks to school.

A) The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:

a) 20

b) 8

c) $\frac{1}{2}$

d) 1

Use words, pictures, or diagrams to explain your answer.

$$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24} \text{ is closest to } 20.$$

Karen's Pre to Post

Sample of Evidence

- 1) Review the student responses in *Karen's pre-assessment*. Which responses include evidence of inappropriate whole number reasoning? What is the evidence?
- 2) Review the student responses in *Karen's post assessment*. To what degree is this error present in the post assessment?
- 3) What is the evidence in Karen's post assessment that suggests a possible instructional focus in Karen's classroom?

Study Work Sub-Study – Fall 2005

(Sample = 19.7% (39/198) of 4th grade pre/post assessments)

Test: 2005 study...

- Evidence of use of inappropriate whole number reasoning
- Use of models to solve problems

Sampled:

2 fourth grade classrooms
(2/8 of classrooms)

Analyzed all pre and post
assessment questions

Preliminary Findings Grade 4 Whole Number Reasoning

Findings

- **38.2%** (129/338) of all students responses reviewed in the **pre-assessment** included evidence of inappropriate use of whole number reasoning;
- **7.4%** (25/338) of all students responses reviewed in the **post-assessment** included evidence of inappropriate use of whole number reasoning;
- Inappropriate use of whole number reasoning was evidenced in **52%** (129/247) of the errors in the pre-assessment while only **22.1%** (25/113) in the post assessment.

Code for Student Generated Models

Three questions were asked:

- **Did the student use a model to help solve the problem?**
- **What type of model? (area, set, or linear)**
- **Was the model used effectively?**

Preliminary Findings Grade 4 (n = 39 students)

- 23.1% (9/39) of the students **effectively used** one or more models in the pre-assessment
- 79.5% (31/39) of the students in the sample **effectively used** one or more models in the post assessment while only.
- 50% of the students who used models effectively, used 3 or more models in the post assessment.

The good and bad news

Findings

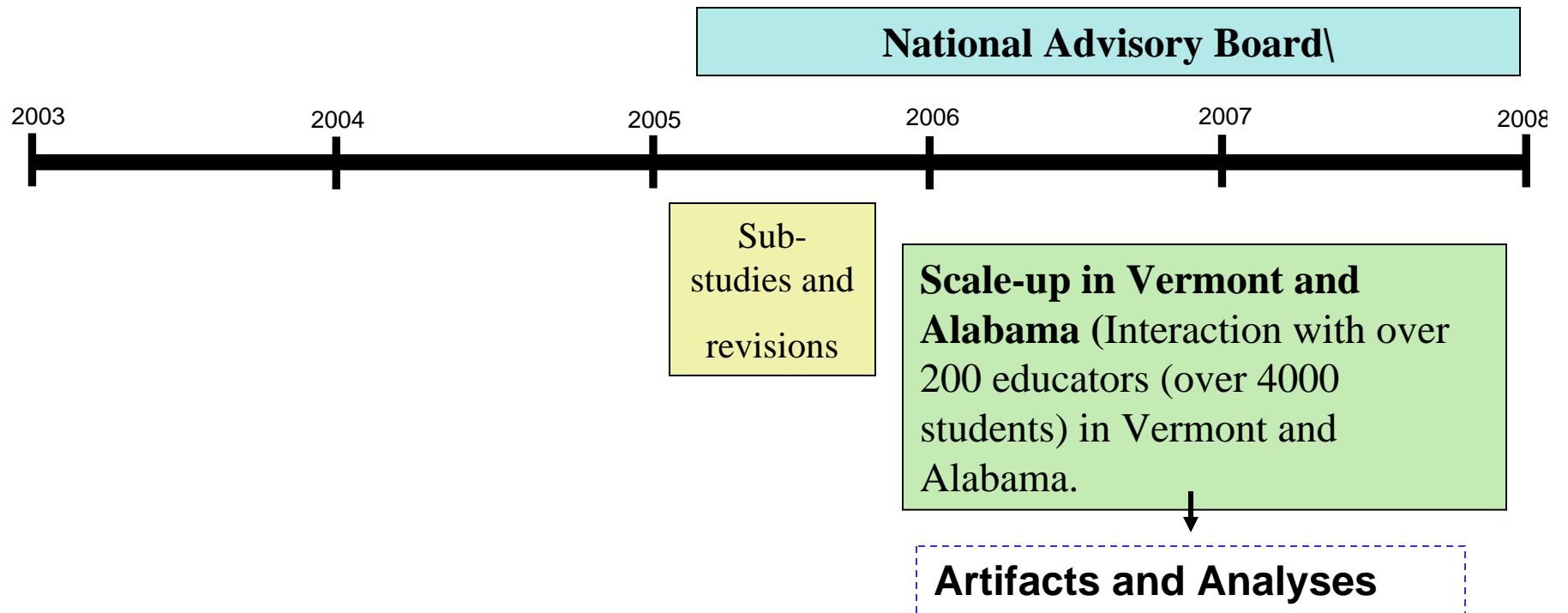
Good News

- Inappropriate whole number reasoning was less evident in post assessment than pre-assessment
- Students were using models --- including an increase in the use of number lines

Bad news

- When 5th and 6th student work was reviewed – models were still the dominant strategy to solve problems like comparing and ordering fractions.

Design Committee – school based leaders and teachers, assessment expert, a mathematician
(distillation of hundreds of research articles used as the foundation of OGAP tools and resrouces0)



From this we increased the emphasis in PD materials

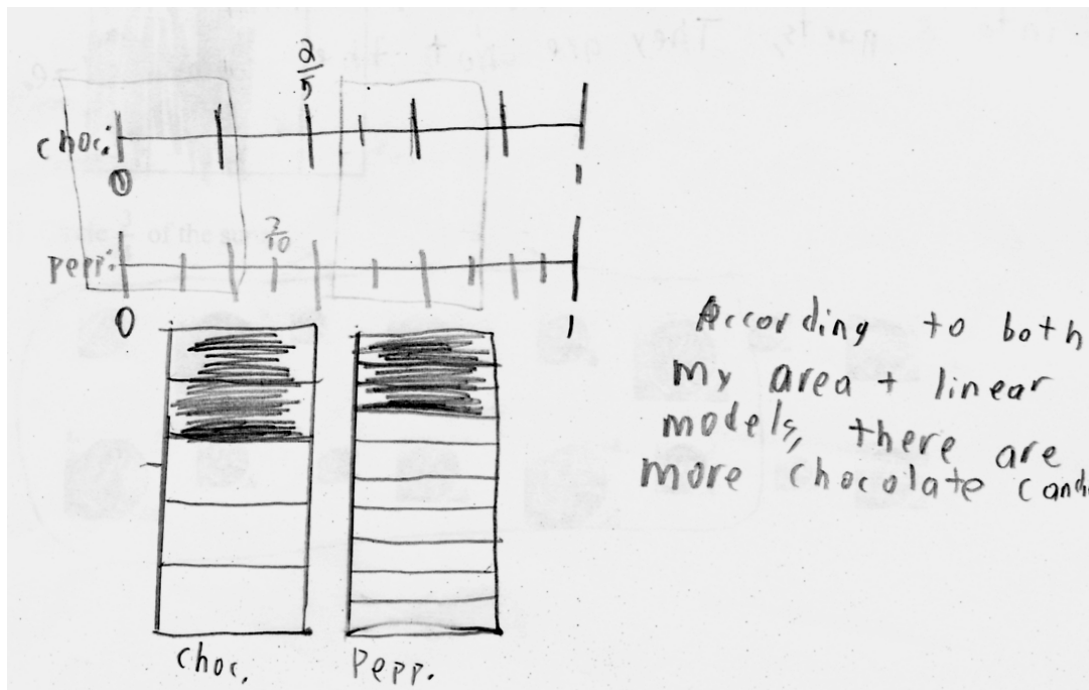
- “Models as a means to the mathematics, not the ends.”
- Use of other reasoning strategies...

There are some candies in a dish.

$\frac{2}{5}$ of the candies are chocolate.

$\frac{3}{10}$ of the candies are peppermint.

Are there more chocolate candies or more peppermint candies?



Revision: Explicitly engaged teachers in cases/activities that helped them understand how to help students move from models as the “means to the mathematics” not the ends?

To help Mr. Laird please answer the following:

- 1) What understandings are evidenced in Mathew's work? Describe.
- 2) How could these evidences be capitalized on to build understanding about equivalence and common denominators when comparing fractions, or adding and subtracting fractions?

Researchers found that students effectively used five types of reasoning when solving problems involving fractions: (*Behr, M., & Lesh, R. (1992)*)

- o Unit fraction reasoning
- o Extended unit fraction reasoning
- o Reference points
- o Models (manipulatives or drawn)
- o Common denominators

Revision: Changed the professional development materials to promote a use of range of strategies for solving problems involving fractions.

Identify fraction pairs or sets that provide the opportunity for different types of reasoning.

1. $\frac{3}{6}$ $\frac{5}{6}$

4. $\frac{3}{6}$ $\frac{7}{15}$

7. $\frac{31}{64}$ $\frac{37}{50}$

2. $\frac{11}{13}$ $\frac{9}{11}$

5. $\frac{1}{7}$ $\frac{1}{5}$

8. $\frac{8}{25}$ $\frac{15}{50}$

3. $\frac{7}{9}$ $\frac{7}{11}$

6. $\frac{15}{38}$ $\frac{5}{13}$

9. $\frac{8}{9}$ $\frac{10}{11}$

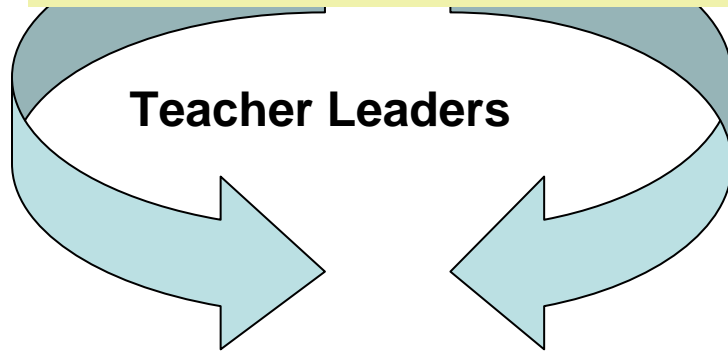
Principles for Scaling-up...

... based on findings from the 2005 Exploratory Study and recommendations of OGAP National Advisory Board

- **Capitalize on teacher leadership**
- **Provide professional development...**
 - about formative assessment in general, but specific to a mathematical topic;
 - in the use of OGAP processes and materials; and
 - on the cognitive research that underpins OGAP processes and materials.
- **Provide resources and support materials necessary for effective implementation.**
- **Provide mentor support during implementation.**

OGAP Fraction Scale-up – Capitalizing on Teacher Leadership

Teacher Leaders - 4 credit course with year long mentor support



Phase I: Your learning and experience

Phase II: Supporting mentees



Mentees – 12 hours of PD with mentor support when using OGAP materials and resources

Test: Is there evidence that teachers use a range of strategies when they solve problems and that their instruction focuses on using modeling as “ means not an end”?

Artifacts

- Unit plans (teacher leaders)
- Teacher action research
- Post Surveys
- Teacher background surveys
- **Pilot teacher assessment**

Provide three strategies students can use to solve this problem.
Provide examples.

1) Which fraction is closest to 1? Show your work.

$$\frac{1}{2}$$

$$\frac{7}{9}$$

$$\frac{11}{13}$$

$$\frac{1}{6}$$

Pilot OGAP Teacher
Assessment Questions

Pre-assessment Q1 A

① $\frac{1}{2} = \frac{117}{234}$ $\frac{7}{9} = \frac{182}{234}$ $\frac{11}{13} = \frac{198}{234}$
 $\frac{1}{6} = \frac{39}{234}$ $\therefore \frac{11}{13}$ is closest to 1

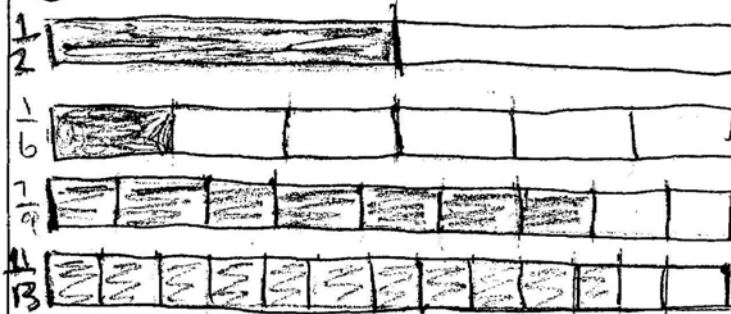
② Use fraction bars kit provided, (ninths + thirteenths are in it).

③

Post-assessment Q1 A

① Unit fractions: $\frac{1}{2}, \frac{1}{6}$
 sixths are smaller parts than halves.

② Use of area models



③ Use $\frac{1}{2}$ benchmark.
 Using unit fraction reasoning, $\frac{1}{6}$ is smaller than $\frac{1}{2}$.
 $\frac{7}{9}$ and $\frac{11}{13}$ are greater than $\frac{1}{2}$.
 (continue on back as needed)

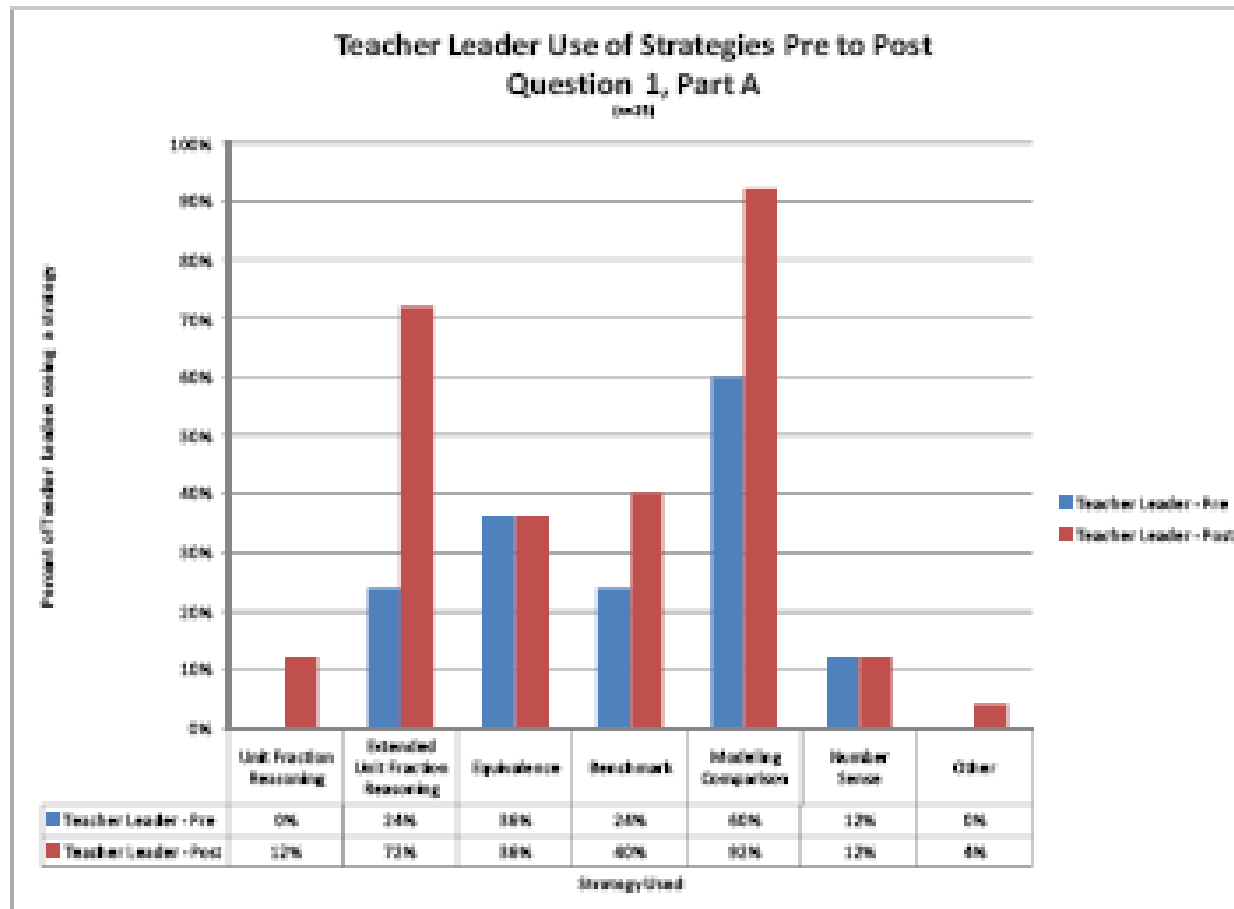
$\frac{11}{13}$ is $\frac{2}{13}$ away from 1 whole.
 $\frac{7}{9}$ is $\frac{2}{9}$ away from the 1 whole.
 Since 13ths are smaller, $\frac{11}{13}$ is closer to 1.

Preliminary (12 points possible)

Mentors and Mentees Pre - Post Teacher Assessment			
	Pre mean	Post mean	T-test (p-) Significance ($p < 0.05$)
Mentors (n=25)	6.16	9.8	3.52E-08
Mentees (n= 42)	5.6	7.9	7.73E-06

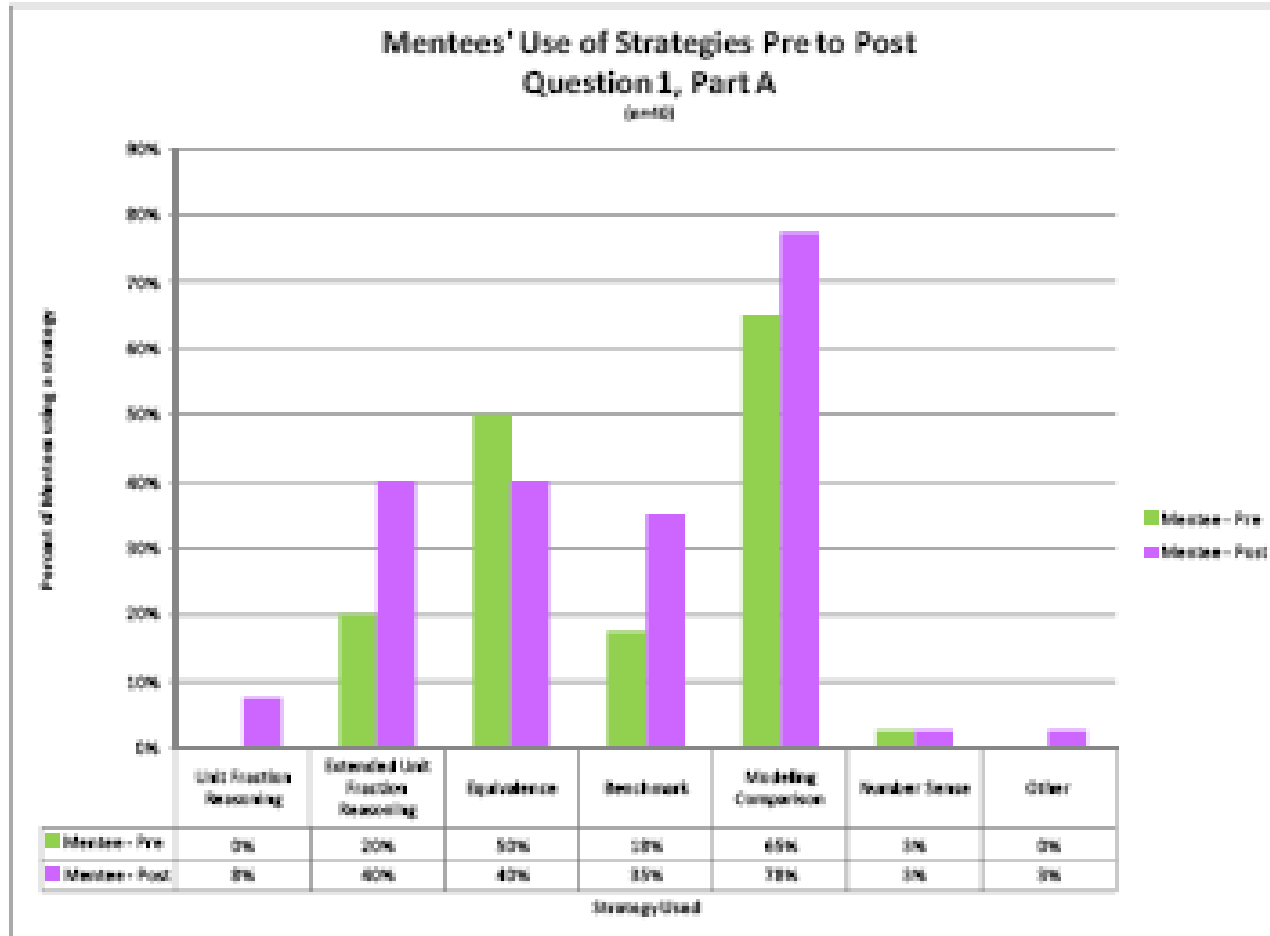
Teacher Pre-Post Preliminary Data – March 2008

Research question – Did teachers increase the range of strategies that they used to solve the problems? **Findings**



Teacher Pre-Post Preliminary Data (March 2008)

Evidence



Data suggests that ---

- Teacher leaders increased the range of strategies that they used pre to post to solve the two problems.
- Mentees also increased the range, but to a lesser degree.

Given these data what are some potential next steps for revision ---

- What are additional research questions?
- What are other sources that have the potential to inform these questions?

Artifacts

- Unit plans (teacher leaders)
- Teacher action research
- Post Surveys
- Teacher background surveys
- Pilot teacher assessment
- Advisory Board

For more information...

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