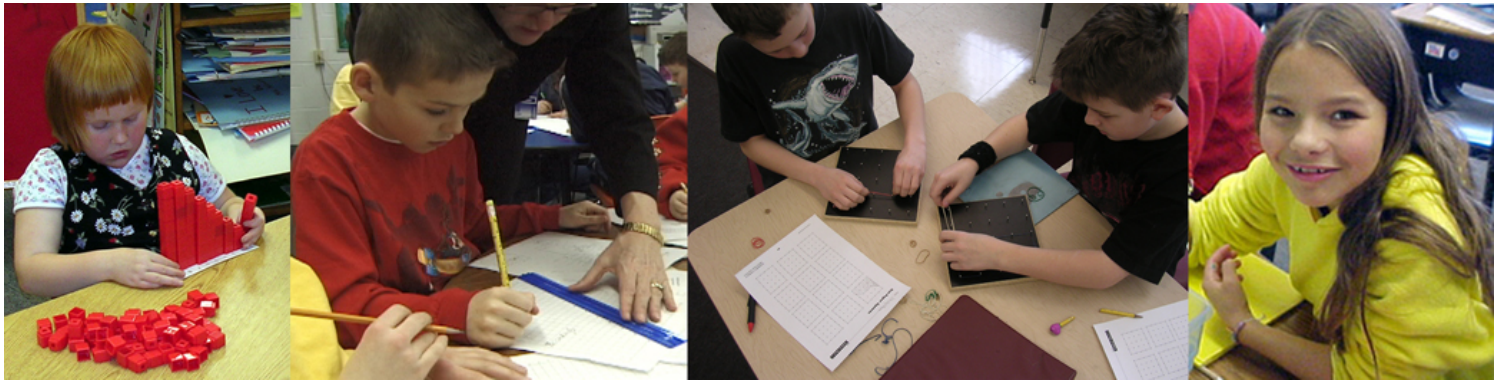

Facilitating Use of Formative Assessments: Multiplicative Reasoning– Ongoing Assessment Project

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Marge Petit, Marge Petit Consulting, MPC (mpetit@gmavt.net)

OGAP Sites:

Vermont
Alabama
Michigan
Ohio
Amman, Jordan
Nebraska

THE VERMONT INSTITUTES
Equity and Excellence for all Students

MPC Educational Consulting

Vermont Mathematics Partnership

2012 NCSM Annual Meeting

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In the end – it is the evidence of student thinking not just from assessment questions, but also from classroom discussions and activities that informs instructional decision making.

Take Aways!

- **Teacher knowledge** about the research/learning trajectories is fundamental – this involves a real commitment to PD, NOT just creating tools and materials, but substantive professional development.
- **Evidence of Student Thinking** - it is the evidence of student thinking not just from assessment questions, but from classroom discussions and activities that informs instructional decision making.
- **Formative assessment** is a powerful tool when it is implemented systematically and intentionally coupled with the above.
- The **CCSS** and OGAP Framework

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In 1 hour…

What can be done ….

- …provide participants with the big idea of OGAP and introduce to the OGAP Multiplicative Framework

What <u>cannot</u> be done…

- … provide participants with a deep understanding of the details and potential implications of OGAP and the research related to students developing their multiplicative reasoning
- …be sure that participants understand the difference between formative and summative assessment.

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OGAP is a systematic and intentional formative assessment system in mathematics.

- Gathering information about pre-existing knowledge through the use of a **pre-assessment**;
- **Analysing pre-assessment** to guide unit planning; and
- **A continuous and intentional system** of instructing, probing with instructionally embedded questions, analysis, and instructional modification.

Grades 2 - 8

- Fractions
- Multiplicative reasoning
- Proportionality

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In place and in use for all 3 mathematical topics

- Pre-assessments and ongoing questions
- Tools and strategies to analyze student work
- Professional development workshop materials and resources to communicate research and support the use of OGAP formative assessment system

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OGAP was Developed Based on Four Principles

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Principle # 1: *Build on pre-existing knowledge* (*How People Learn (2000)* National Research Council)

Principle # 2*: Learn (and assess) for Understanding*

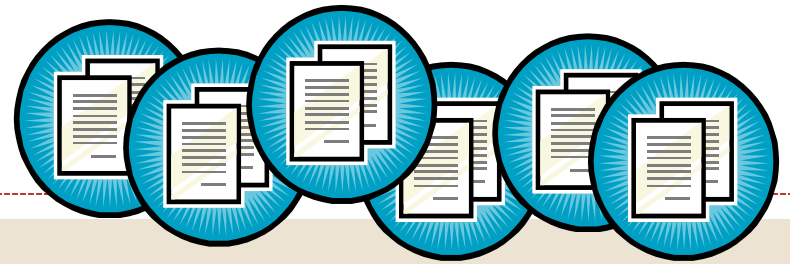
(Adding it Up! (*2001) National Research Council*)

Principle # 3: *Use Frequent Formative Assessment*
(Inside the Black Box, (2001) Black, P, and Wiliam, D.)

Principle # 4: *Build Assessment on Mathematics Education Research* *(Knowing What Students Know (2001) National Research Council)*

It is not formative assessment alone OR knowledge of cognitive research alone…

…but the marriage of the two that empowers teachers



Hundreds of research articles distilled into a frameworks and used

In design of materials

- formative assessment items
- professional development materials (case studies, activities, essays)
- Book and articles

In work with educators

- analyze student work
- inform instructional decisions
- help understand the purposes of activities in mathematics programs

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Research to Practice

Teachers say understanding the math education research help them…

- Understand the purposes of activities in math programs;
- Understand evidence in student work used to inform instruction;
- Strengthen and focus first wave instruction;
- Respond to evidence in student work as instruction proceeds.

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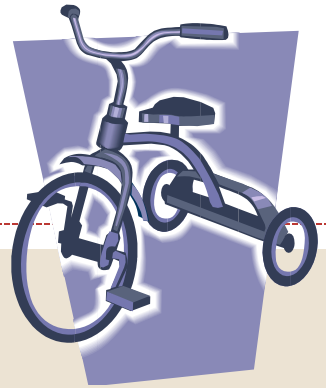
Solve the following problems 3 different ways



A) How many wheels do 5 tricycles have?

B) How many wheels do 29 tricycles have?

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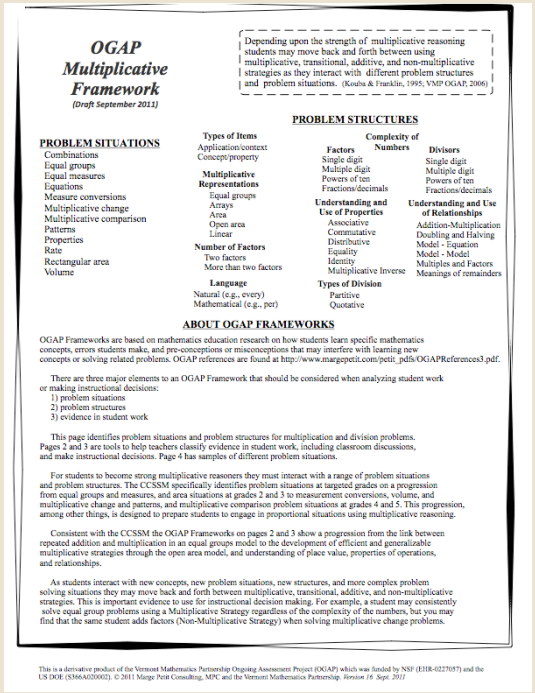
Review the Framework



OGAP Multiplicative Framework
(Draft September 2011)

Depending upon the strength of multiplicative reasoning students may move back and forth between using multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem structures and problem situations. (Kouba & Franklin, 1995; VMP OGAP, 2006)

PROBLEM SITUATIONS
Combinations
Equal groups
Equal measures
Equations
Measure conversions
Multiplicative change
Multiplicative comparison
Patterns
Properties
Rate
Rectangular area
Volume

PROBLEM STRUCTURES

Types of Items
Application/context
Concept/property

Multiplicative Representations
Equal groups
Arrays
Area
Open area
Linear

Number of Factors
Two factors
More than two factors

Language
Natural (e.g., every)
Mathematical (e.g., per)

Complexity of Numbers

Factors
Single digit
Multiple digit
Powers of ten
Fractions/decimals

Understanding and Use of Properties
Associative
Commutative
Distributive
Equality
Identity
Multiplicative Inverse

Types of Division
Partitive
Quotative

Divisors
Single digit
Multiple digit
Powers of ten
Fractions/decimals

Understanding and Use of Relationships
Addition-Multiplication
Doubling and Halving
Model - Equation
Model - Model
Multiples and Factors
Meanings of remainders

ABOUT OGAP FRAMEWORKS

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems. OGAP references are found at http://www.margepetit.com/petit_pdfs/OGAPReferences3.pdf.

There are three major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions:
1) problem situations
2) problem structures
3) evidence in student work

This page identifies problem situations and problem structures for multiplication and division problems. Pages 2 and 3 are tools to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions. Page 4 has samples of different problem situations.

For students to become strong multiplicative reasoners they must interact with a range of problem situations and problem structures. The CCSSM specifically identifies problem situations at targeted grades on a progression from equal groups and measures, and area situations at grades 2 and 3 to measurement conversions, volume, and multiplicative change and patterns, and multiplicative comparison problem situations at grades 4 and 5. This progression, among other things, is designed to prepare students to engage in proportional situations using multiplicative reasoning.

Consistent with the CCSSM the OGAP Frameworks on pages 2 and 3 show a progression from the link between repeated addition and multiplication in an equal groups model to the development of efficient and generalizable multiplicative strategies through the open area model, and understanding of place value, properties of operations, and relationships.

As students interact with new concepts, new problem situations, new structures, and more complex problem solving situations they may move back and forth between multiplicative, transitional, additive, and non-multiplicative strategies. This is important evidence to use for instructional decision making. For example, a student may consistently solve equal group problems using a Multiplicative Strategy regardless of the complexity of the numbers, but you may find that the same student adds factors (Non-Multiplicative Strategy) when solving multiplicative change problems.

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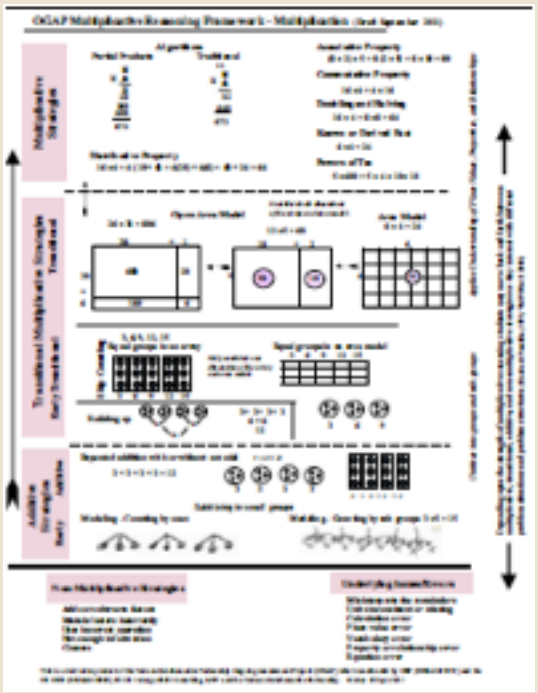

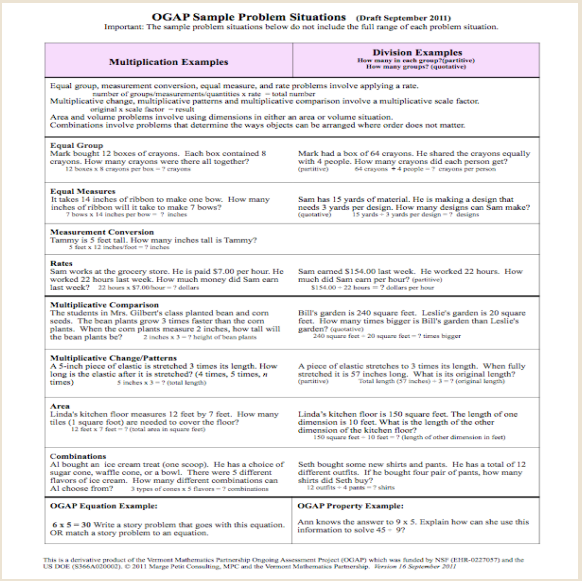

OGAP Sample Problem Situations (Draft September 2011)
Important: The sample problem situations below do not include the full range of each problem situation.

Multiplication Examples	Division Examples How many in each group? (partitive) How many groups? (quotative)
Equal group, measurement conversion, equal measure, and rate problems involve applying a rate. number of groups/measurements/quantities x rate = total number Multiplicative change, multiplicative patterns and multiplicative comparison involve a multiplicative scale factor. original x scale factor = result Area and volume problems involve using dimensions in either an area or volume situation. Combinations involve problems that determine the ways objects can be arranged where order does not matter.	
Equal Group Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there all together? 12 boxes x 8 crayons per box = ? crayons	Mark had a box of 64 crayons. He shared the crayons equally with 4 people. How many crayons did each person get? (partitive) 64 crayons ÷ 4 people = ? crayons per person
Equal Measures It takes 14 inches of ribbon to make one bow. How many inches of ribbon will it take to make 7 bows? 7 bows x 14 inches per bow = ? inches	Sam has 15 yards of material. He is making a design that needs 3 yards per design. How many designs can Sam make? (quotative) 15 yards ÷ 3 yards per design = ? designs
Measurement Conversion Tammy is 5 feet tall. How many inches tall is Tammy? 5 feet x 12 inches/foot = ? inches	
Rates Sam works at the grocery store. He is paid $7.00 per hour. He worked 22 hours last week. How much money did Sam earn last week? 22 hours x $7.00/hour = ? dollars	Sam earned $154.00 last week. He worked 22 hours. How much did Sam earn per hour? (partitive) $154.00 ÷ 22 hours = ? dollars per hour
Multiplicative Comparison The students in Mrs. Gilbert's class planted bean and corn seeds. The bean plants grow 3 times faster than the corn plants. When the corn plants measure 2 inches, how tall will the bean plants be? 2 inches x 3 = ? height of bean plants	Bill's garden is 240 square feet. Leslie's garden is 20 square feet. How many times bigger is Bill's garden than Leslie's garden? (quotative) 240 square feet ÷ 20 square feet = ? times bigger
Multiplicative Change/Patterns A 5-inch piece of elastic is stretched 3 times its length. How long is the elastic after it is stretched? (4 times, 5 times, *n* times) 5 inches x 3 = ? (total length)	A piece of elastic stretches to 3 times its length. When fully stretched it is 57 inches long. What is its original length? (partitive) Total length (57 inches) ÷ 3 = ? (original length)
Area Linda's kitchen floor measures 12 feet by 7 feet. How many tiles (1 square foot) are needed to cover the floor? 12 feet x 7 feet = ? (total area in square feet)	Linda's kitchen floor is 150 square feet. The length of one dimension is 10 feet. What is the length of the other dimension of the kitchen floor? 150 square feet ÷ 10 feet = ? (length of other dimension in feet)
Combinations Al bought an ice cream treat (one scoop). He has a choice of sugar cone, waffle cone, or a bowl. There were 5 different flavors of ice cream. How many different combinations can Al choose from? 3 types of cones x 5 flavors = ? combinations	Seth bought some new shirts and pants. He has a total of 12 different outfits. If he bought four pair of pants, how many shirts did Seth buy? 12 outfits ÷ 4 pants = ? shirts
OGAP Equation Example: **6 x 5 = 30** Write a story problem that goes with this equation. OR match a story problem to an equation.	**OGAP Property Example:** Ann knows the answer to 9 x 5. Explain how can she use this information to solve 45 ÷ 9?

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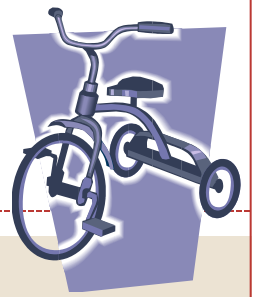
Intro to OGAP Framework: About the problem



- What is the problem situation?
- What are other problem structures to consider?
- What are strategies that were used? Where are they on the OGAP Framework?

> Depending upon the strength of multiplicative reasoning students may move back and forth between using multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem structures and problem situations. (Kouba & Franklin, 1995; VMP OGAP, 2006)

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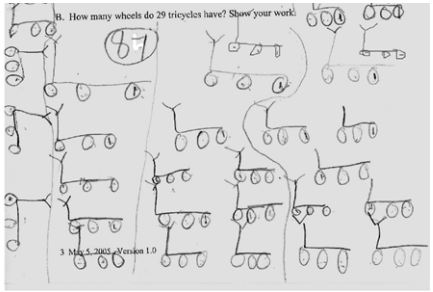
Analyzing evidence

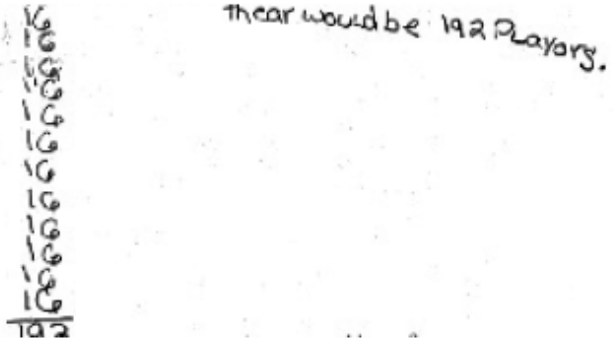
Going beyond celebrating different strategies TO…

…understanding the instructional implications of the strategies and taking action

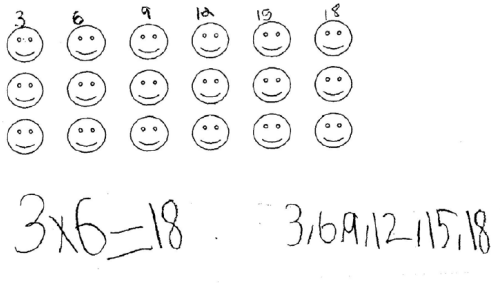
How many wheels do 29 tricycles have?
One tricycle has three wheels.



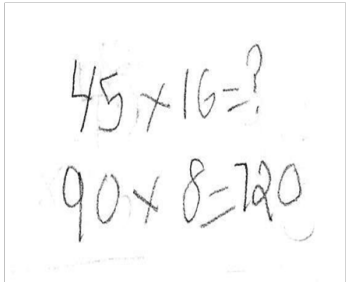
There are 16 players on a team in the Smithville Soccer League. How many players are in the league if there are 12 teams?



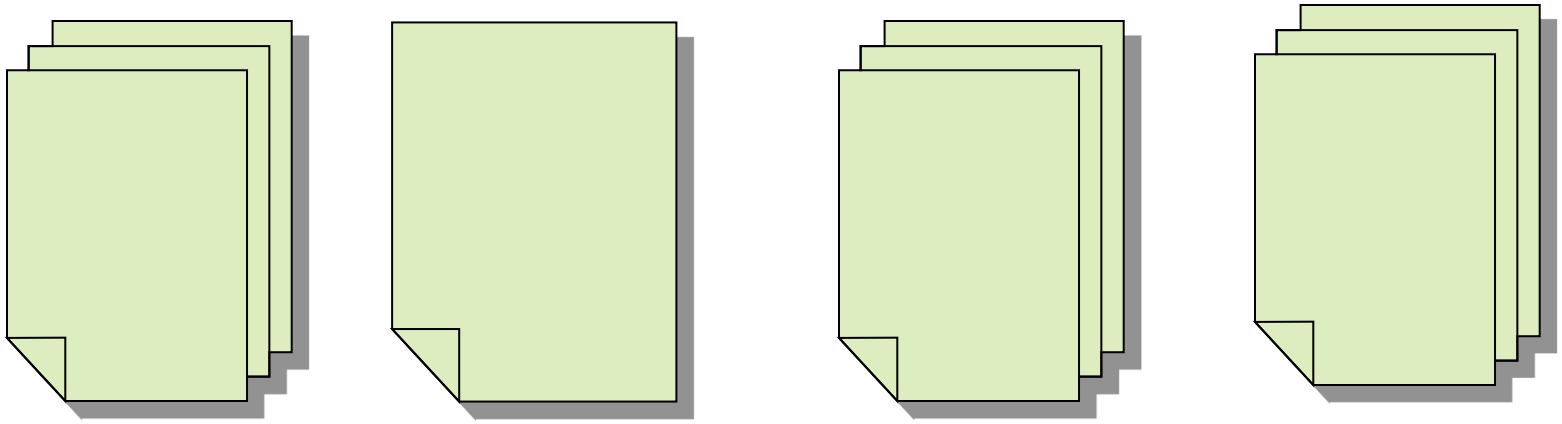
Write an equation to match this picture.



A class has set a goal that each student will read 45 pages this week. There are 16 students in the class. How many pages will they have read altogether by the end of the week?



Analyzing student work – the OGAP Sort



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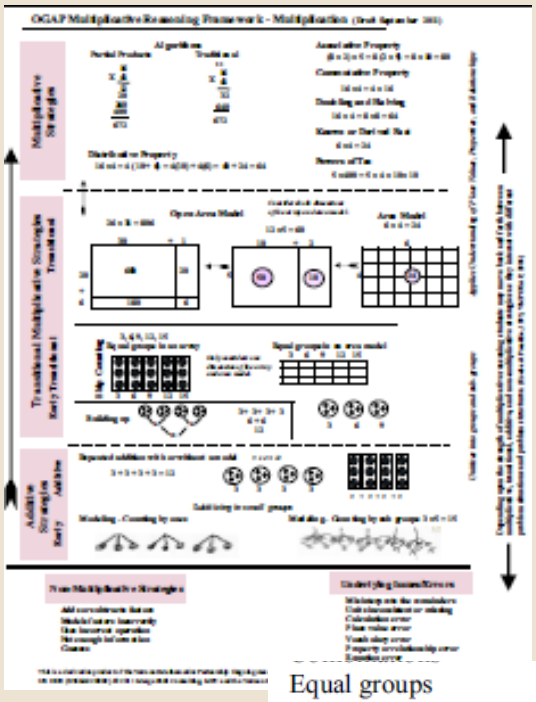
CCSS Whole Number Multiplication Link to the OGAP Framework



Equal groups
Equal measures
Equations
Measure conversions
Multiplicative change
Multiplicative comparison
Patterns
Properties
Rate
Rectangular area
Volume

	Problem Situations	Strategies
Grade 2	Equal groups	Repeated addition
Grade 3 (factors within 100)	Equal groups, arrays, equal measures, beginning area	Properties of operations, drawings, equations
Grade 4 (1 digit x 4 digit, and 2 digit x 2 digit)	Multiplicative comparison, measurement conversion within systems, area	Place value and properties… using equations, rectangular arrays, and/or area model, equations
Grade 5 (fluently)	Scaling (multiplicative change), area, volume, patterns, conversions between systems	Standard algorithm, equations

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Grades 2 - 8

- Fractions
- Multiplicative reasoning
- Proportionality

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Questions and Answers

What do *teacher leaders and teachers* say about their experience in relationship to the stated goals and the use of OGAP formative assessment system?

Results based on a spring 2007 online survey

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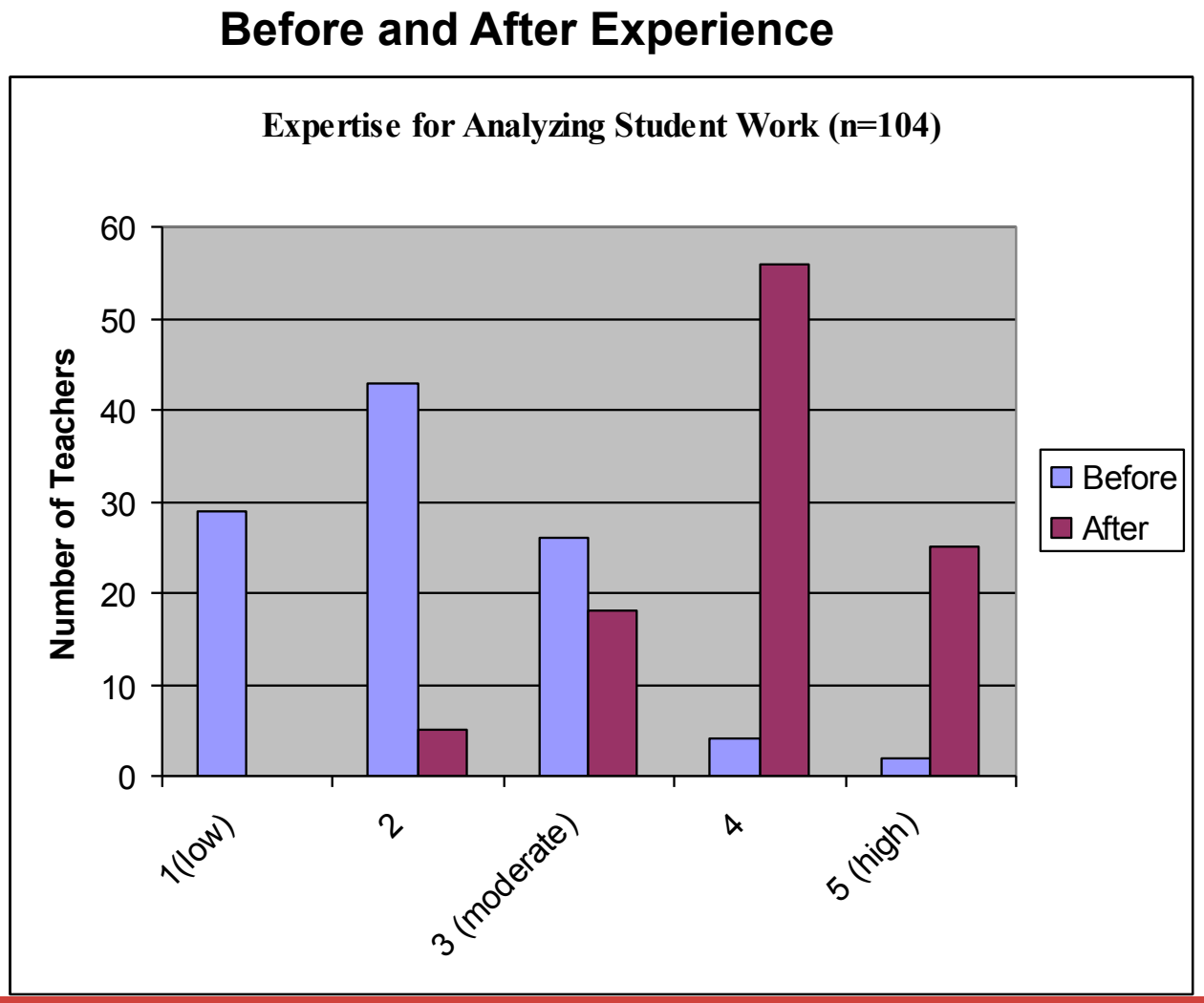
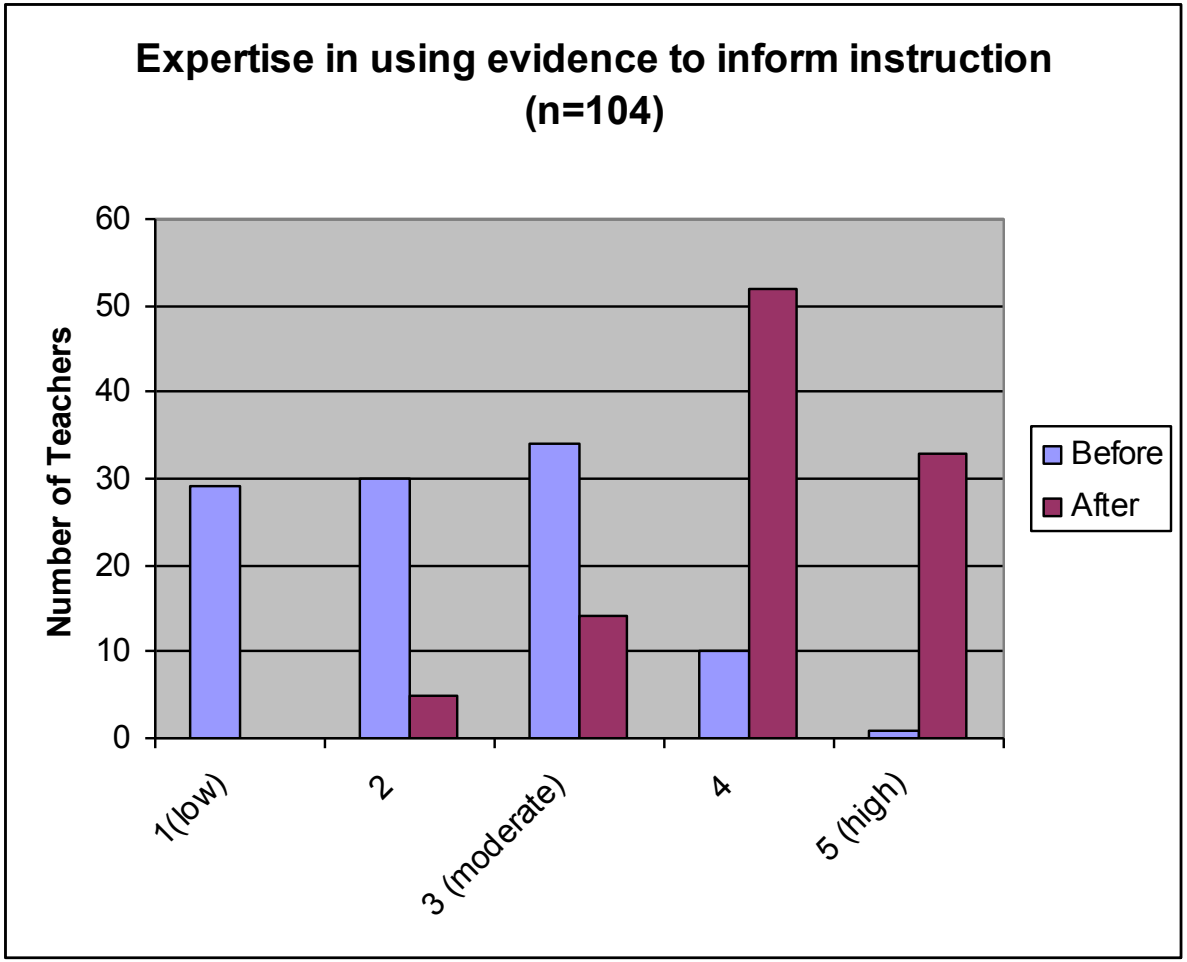
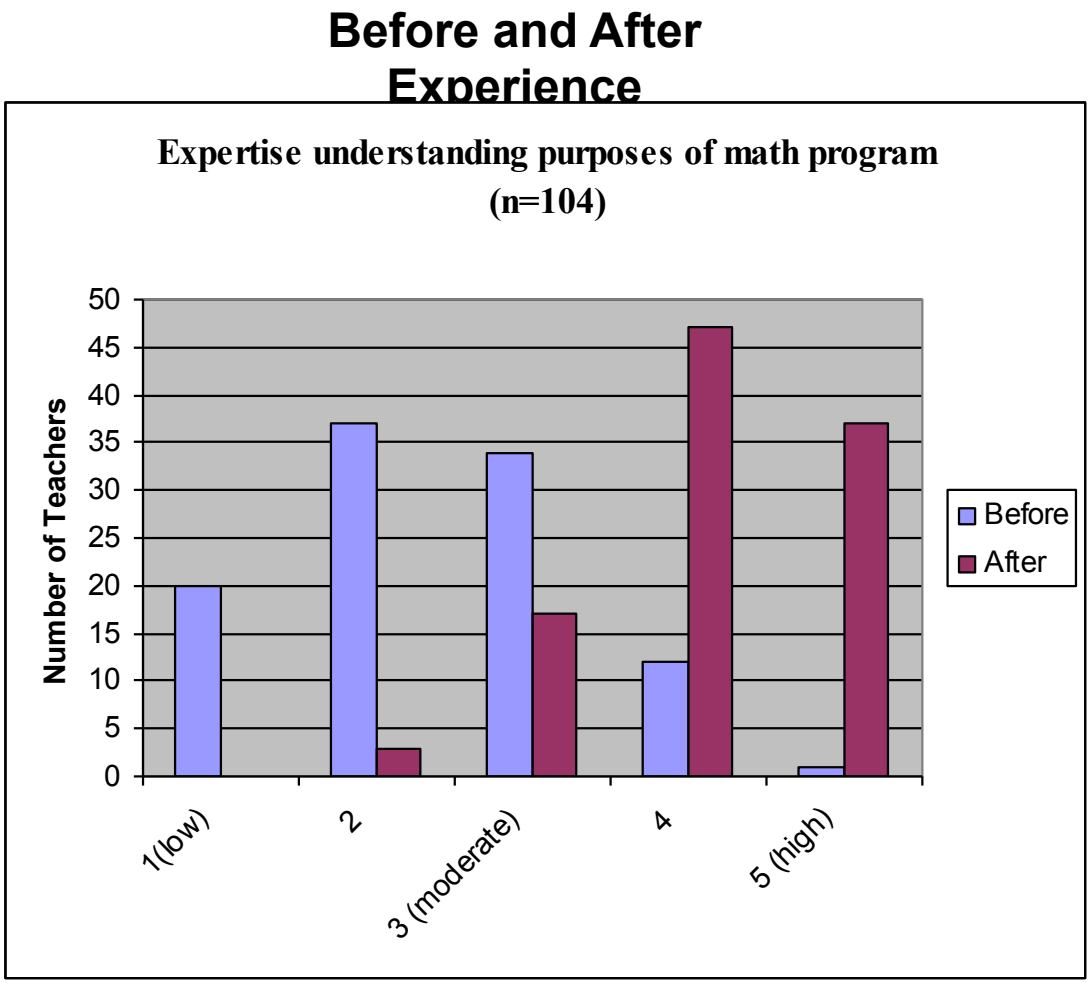
Expertise for analyzing student work (for evidence of developing understanding, common errors and misconceptions)…



Expertise in using evidence in student work to inform instruction…

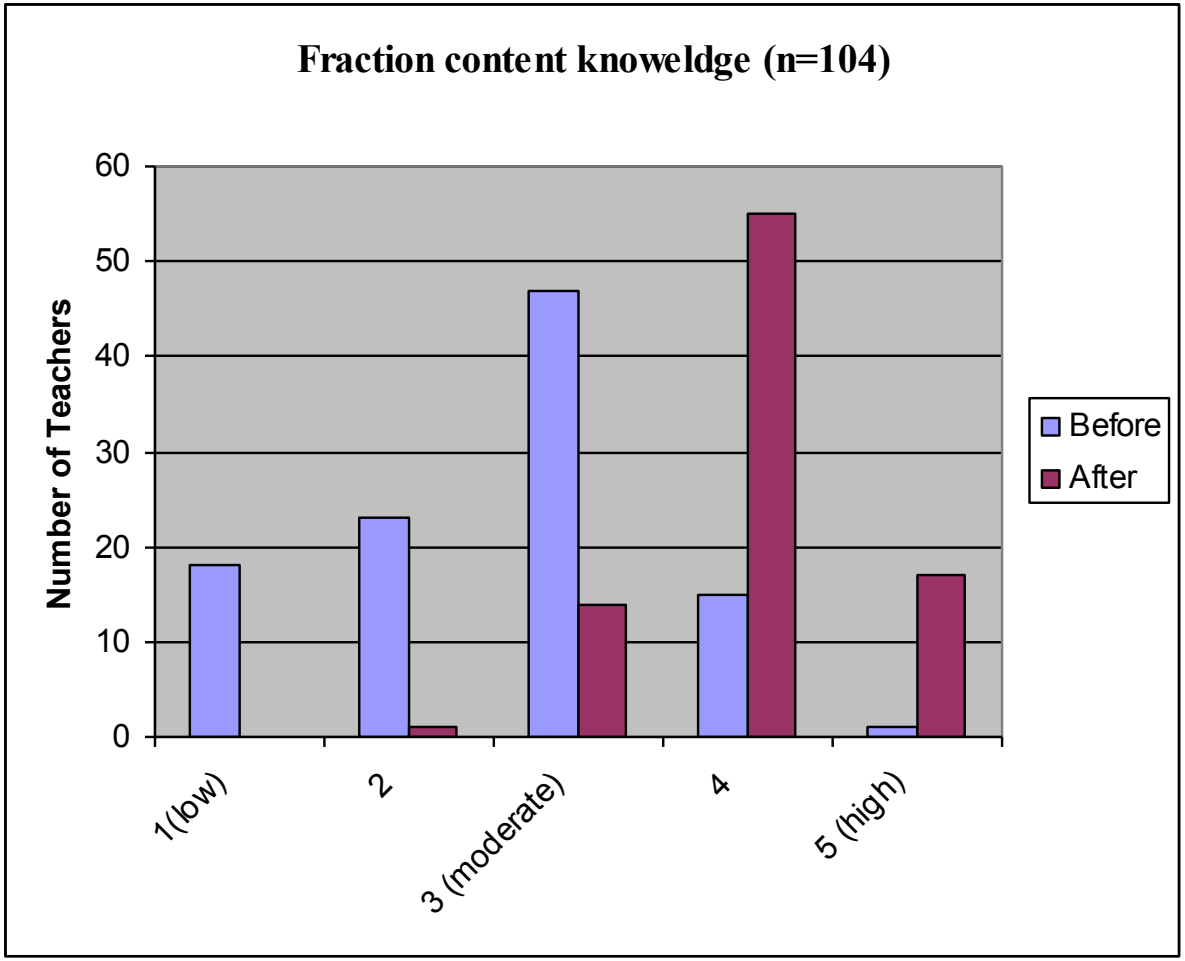


Understanding purposes of activities in mathematics program…



Fraction content knowledge…

Before and After Experience



Pre-post Question – Pilot OGAP Teacher Assessment (2007)

Provide three strategies students can use to solve this problem. Provide examples.

1) Which fraction is closest to 1? Show your work.

$\frac{1}{2}$ $\frac{7}{9}$ $\frac{11}{13}$ $\frac{1}{6}$

Pilot OGAP Teacher Assessment Question

Sample Teacher Responses

Pre-assessment Q1 A

① $\frac{1}{2} = \frac{117}{234}$ $\frac{7}{9} = \frac{182}{234}$ $\frac{11}{13} = \frac{198}{234}$

$\frac{1}{6} = \frac{39}{234}$ ∴ $\frac{11}{13}$ is closest to 1

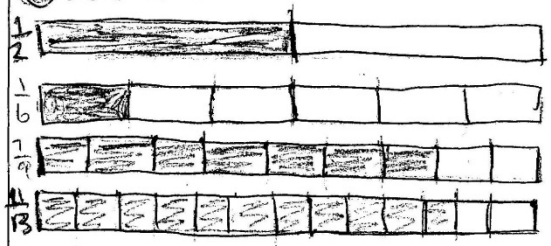
② Use fraction bars kit provided, (ninths + thirteenths are in it.)

③

Post-assessment Q1 A

① Unit fractions: $\frac{1}{2}$, $\frac{1}{6}$
sixths are smaller parts than halves

② Use of area models



③ Use $\frac{1}{2}$ benchmark.
Using unit fraction reasoning, $\frac{1}{6}$ is smaller than $\frac{1}{2}$.
$\frac{7}{9}$ and $\frac{11}{13}$ are greater than $\frac{1}{2}$.
(continue on back as needed)

$\frac{11}{13}$ is $\frac{2}{13}$ away from 1 whole.
$\frac{7}{9}$ is $\frac{2}{9}$ away from the 1 whole.
Since 13ths are smaller, $\frac{11}{13}$ is closer to 1.

Findings (Petit-Cunningham, 2008)

- Teacher leaders increased the range of strategies that they used pre to post to solve the two problems.
- Mentees also increased the range, but to a lesser degree

Mentors and Mentees Pre - Post Teacher Assessment			
	Pre mean	Post mean	T-test (p-) Significance (p< 0.05)
Mentors (n=25)	6.16	9.8	3.52E-08
Mentees (n= 42)	5.6	7.9	7.73E-06

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Related Publications

- Petit, Laird, and Marsden (2010), A Focus on Fractions: Brining Research to the Classroom. Routledge, New York and London.
- Petit, Laird, & Marsden (September, 2010). They get fractions as pies – but now what?. Mathematics in the Middle School, NCTM, Reston, Virginia.
- Petit, Zawojewski (2010). Formative Assessment in Elementary Classrooms. Teaching and Learning Mathematics: Translating Research for Elementary School Teachers. NCTM, Reston, VA.
- Petit, Zawojewski, Labaddo (2010). Formative Assessment in the Secondary School Classroom. Teaching and Learning Mathematics: Translating Research for Secondary School Teachers. NCTM, Reston, VA.
- Petit in Daro, Mosher, & Corcoran. (2011). Going from research to practice: Learning trajectories in action. *Mathematics Learning Trajectory Report*. Consortium for Policy Research in Education: Teacher' s College, Columbia University (pp. 35-39). http://www.cpre.org/sites/default/files/researchreport/1220_learningtrajectoriesinmathcciireport.pdf
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- Ercole, Frantz, and Ashline (April 2011). Multiple Ways to Solve Proportional Reasoning Problems. Mathematics Teaching in the Middle School, 16:8, 482-4.

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For OGAP References go to….

- http://margepetit.com/

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Bringing OGAP to your school, district, or state involves…

Significant Professional Development by OGAP team and ongoing support system at the school level

- In an understanding of formative assessment
- In the use of OGAP formative assessment materials and processes.
- on the substance of the math education research that is foundational to the OGAP materials and processes.
- Use of the materials "real time" with students with links to mathematics programs.

Tools and Resources to support system

- Some pre-assessments and ongoing items
- Strategies and related tools for analyzing student work and making instructional decisions

OGAP Development Team and National Advisory Board

Vermont OGAP Design Team

- Leslie Ercole, VMP
- Linda Gilbert, Dotham Brook School
- Kendra Gorton, Milton Elementary School
- Steph Hockenbury, Chamberlin School
- Beth Hulbert, Barre City Elementary and Middle School
- Amy Johnson, Milton Elementary School
- Bob Laird, VMP
- Ted Marsden, Norwich University
- Karen Moylan, Former VMP
- Cathy Newton, Dotham Brook School
- Susan Ojala, Vermont Mathematics Initiative
- Nancy Pollack, Chittenden East
- Marge Petit, Marge Petit Consulting, MPC
- Regina Quinn, VMP
- Loree Silvis, VMP
- Krisan Stone, VMP
- Corrie Sweet, Former VMP
- Tracy Thompson, Ottauquechee School
- Jean Ward, Bennington Rutland Supervisory Union
- Rebecca Young, Hardwick Schools

Plus about 250 Vermont and Alabama teachers and teachers and about 5000 students who participated in OGAP Exploratory Studies and 2006-2008 scale-up

OGAP National Advisory Board

- **Mary Lindquist,** Callaway Professor of Mathematics Education, Emeritus; Past President of the National Council of Teachers of Mathematics
- **Ed Silver**, University of Michigan
- **Judith Zawojewski**, Illinois Institute of Technology

OGAP Sites:

Vermont
Alabama
Michigan
Ohio
Amman, Jordan
Nebraska

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