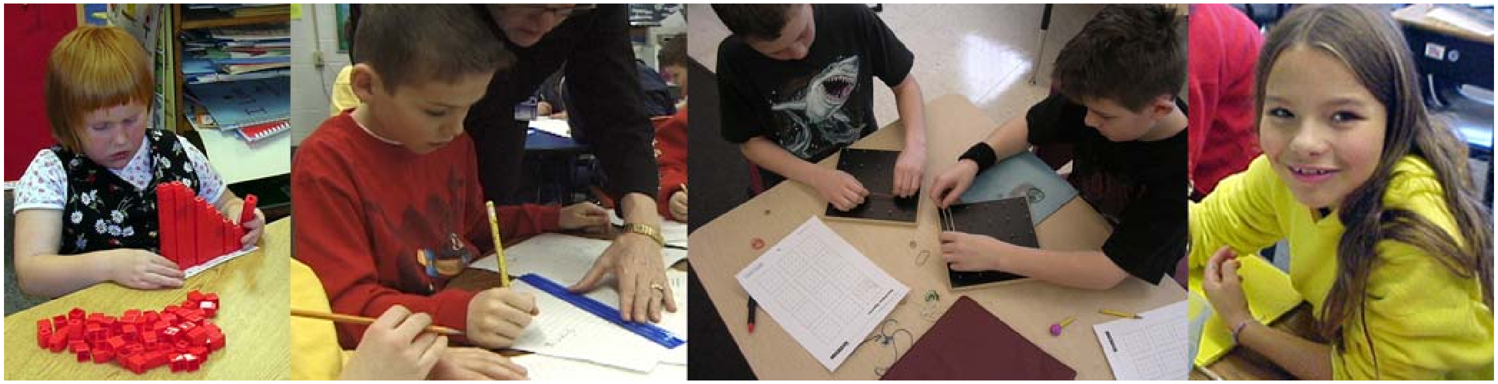

Facilitating Use of Formative Assessment: A Case of Research to Practice

2008 NCSM Annual Meeting

Bob Laird, Vermont Mathematics Institute, University of Vermont (rlaird@uvm.edu)

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THE VERMONT INSTITUTES
Equity and Excellence for all Students

Vermont Mathematics Partnership
www.vermontmathematics.org

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VMP OGAP Design Team

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- **Amy Johnson**, Milton Elementary School
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- **Cathy Newton**, Dotham Brook School
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- **Krisan Stone**, VMP
- **Corrie Sweet**, Former VMP
- **Tracy Thompson**, Ottauquechee School
- **Jean Ward,** Bennington Rutland Supervisory Union
- **Rebecca Young**, Hardwick Schools

Plus about 3000 Vermont and Alabama teachers and teachers and about 6000 students who participated in OGAP Exploratory Studies and 2006-2008 scale-up

Active OGAP National Advisory Board

- **Mary Lindquist,** Callaway Professor of Mathematics Education, Emeritus; Past President of the National Council of Teachers of Mathematics
- **Ed Silver**, University of Michigan
- **Judith Zawojewski**, Illinois Institute of Technology

Goals of Session

- Provide an overview of OGAP materials and processes
- Illustrate some ways that cognitive research permeates OGAP materials and processes
- Provide examples of scale-up models

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The VMP Ongoing Assessment Project (OGAP) was developed to respond to two needs:

1) To improve student learning in mathematics for all students as it relates to Vermont Grade Level Expectations and national standards; and

2) For teachers to obtain quality instructional information as students are developing their understanding of concepts so that interventions for a class as a whole or for individuals can be made "on time."

These needs are shared across the country, not just in Vermont.

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OGAP is an intentional and systematic approach to formative assessment in mathematics involving:

- Gathering information about pre-existing knowledge through the use of a **pre-assessment**;
- **Analysis of pre-assessment** to guide unit planning; and
- **A continuous and intentional system** of instructing, probing with instructionally embedded questions, analysis, and instructional modification.

Grades 2 - 8

- **Fractions**
- **Multiplicative reasoning**
- **Proportionality**

Sources of Evidence and Interactions

Design Committee – school based leaders and teachers, assessment expert, a mathematician (distillation of hundreds of research articles used as the foundation of OGAP tools and resrouces0

Cognitive Labs | revisions | 2004 Study | revisions | 2005 Study

National Advisory Board

200 | 2004 | 2005 | 2006 | 2007 | 2008

Sub-studies and revisions

Exploratory Studies
Purpose: to refine tools, and processes, OGAP professional development.

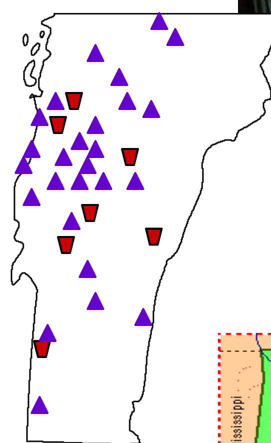
Scale-up in Vermont and Alabama (Interaction with over 200 educators (over 4000 students) in Vermont and Alabama.

Artifacts and Analyses
- Mentor observations
- Student work archives (over 30,000 pieces)
- Teacher logs linked to student work
- Post Surveys
- Interviews
- Teacher background surveys
- Post focus forum
- Student retention study (8 months later)
- Follow-up surveys (8 months later)

Artifacts
- Materials feedback
- Samples of student work
- Unit plans (teacher leaders)
- Teacher action research
- Post Surveys
- Teacher background surveys
- Pilot teacher assessment
- Advisory Board

Development and Implementation Model Based on…

- Distillation of hundreds of research articles and synthesis of research into frameworks
- Interaction with over 300 educators (over 6000 students) in Vermont and Alabama with OGAP materials and resources (and numbers are growing)
- Analysis of teacher logs linked to student work archives
- Analysis of over 40,000 pieces of student work
- Analysis of teacher action research projects
- Surveys, interviews, feedback forms
- Advice from National Advisory Board



Maps not to scale

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In Place and In Use
for all three mathematical topics

- Item banks and pre-assessments
- Tools and strategies to analyze student work
- Professional development workshop materials and resources to communicate research and support the use of OGAP formative assessment system

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OGAP was Developed Based on Four Principles

Principle # 1: Build on pre-existing knowledge

(*How People Learn (2000)* **National Research Council**)

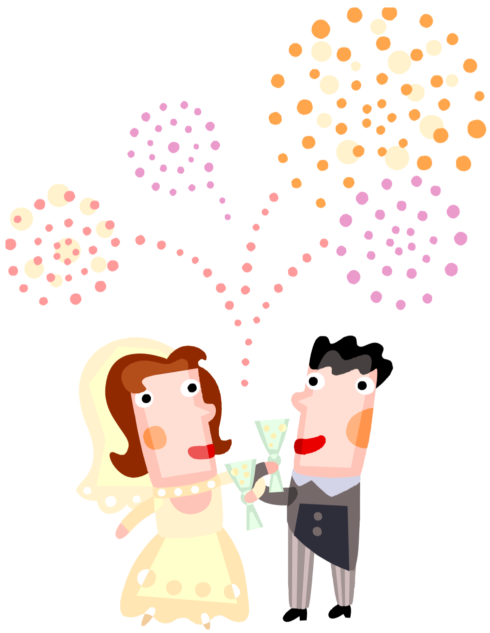
Principle # 2: Learn (and assess) for Understanding (**Adding it Up!** (*2001) National Research Council*)

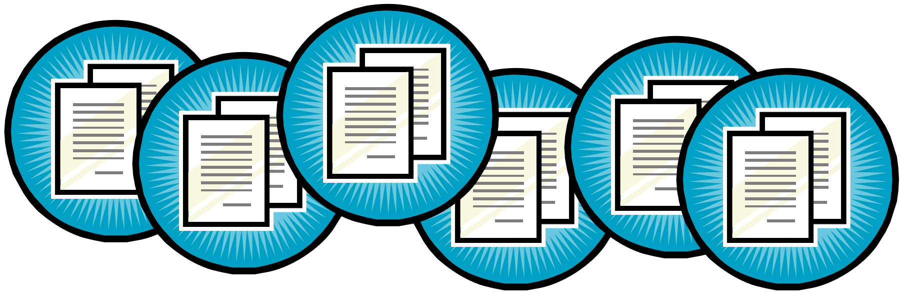
Principle # 3: Use Frequent Formative Assessment *(Inside the Black Box, (2001) Black, P, and Wiliam, D.)*

Principle # 4: Build Assessment on Cognitive Research *(Knowing What Students Know (2001) National Research Council)*

It is not formative assessment alone OR knowledge of cognitive research alone…

…but the marriage of the two that empowers teachers





Hundreds of research articles distilled into a frameworks and used

In design of materials

- formative assessment items (hundreds)
- professional development materials (case studies, activities, essays)

In work with educators

- analyze student work
- inform instructional decisions
- help understand the purposes of activities in mathematics programs

In work with educators

Research to Practice

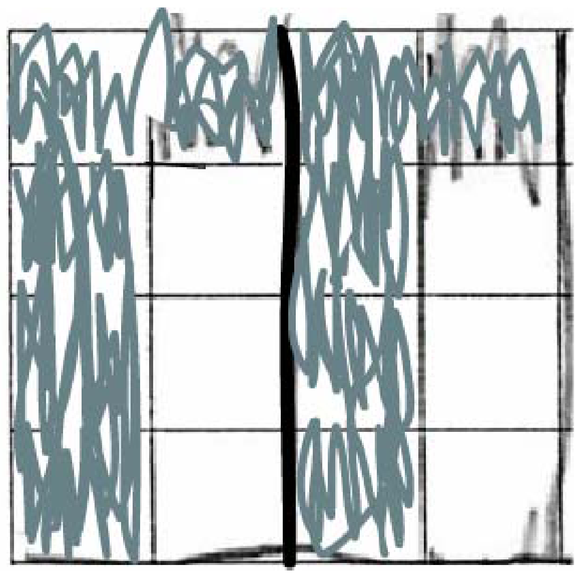
Teachers say – that knowledge of cognitive research coupled with tools and resources sensitive to the research helps them …

- Understand the purposes of activities in math programs;
- Understand evidence in student work used to inform instruction;
- Strengthen and focus first wave instruction;
- Respond to evidence in student work as instruction proceeds.

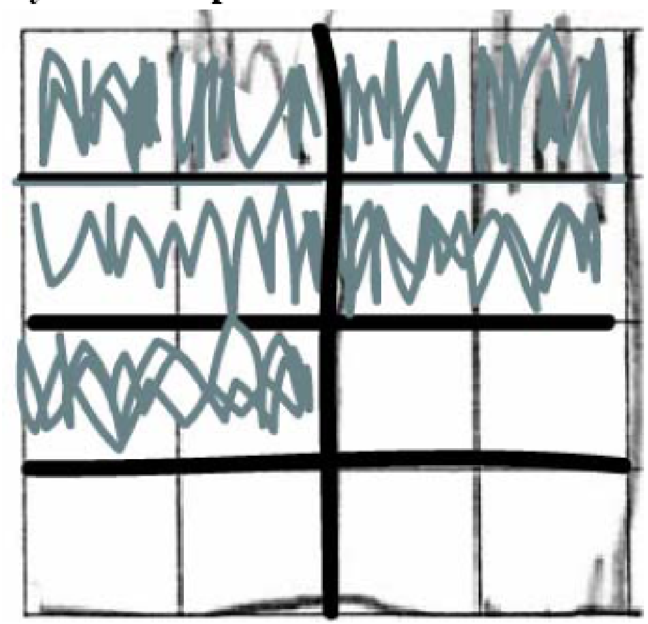
Understand evidence in student work used to inform instruction

Shade $\frac{5}{8}$ of the figure.

Thomas's Response



Dyson's Response



Going beyond celebrating different strategies TO…

…understanding the instructional implications of the strategies and taking action

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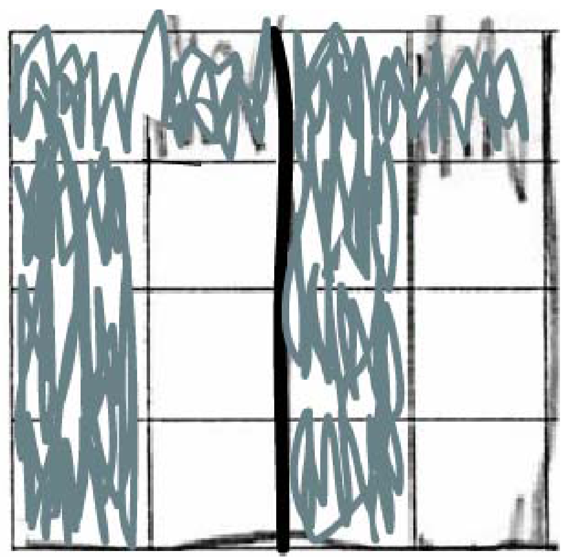
1) How are Dyson’s and Thomas’s responses alike and how are they different?

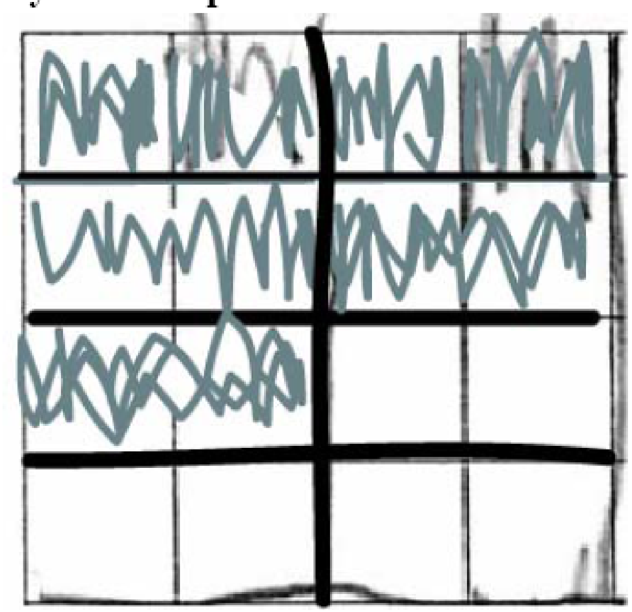
2) Explain to Mr. Purple, using evidence from these pieces of student work, why the strategy a student uses to solve a problem matters even when they get a correct answer.

Shade $\frac{5}{8}$ of the figure.

Thomas’s Response



Dyson’s Response



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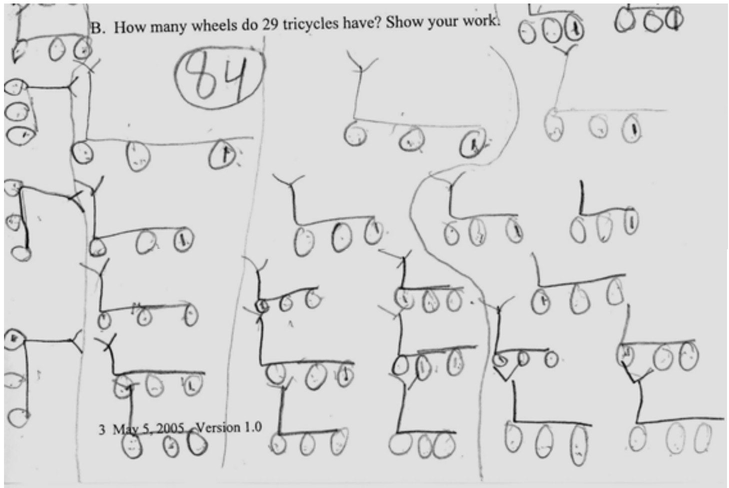
One tricycle has three wheels.

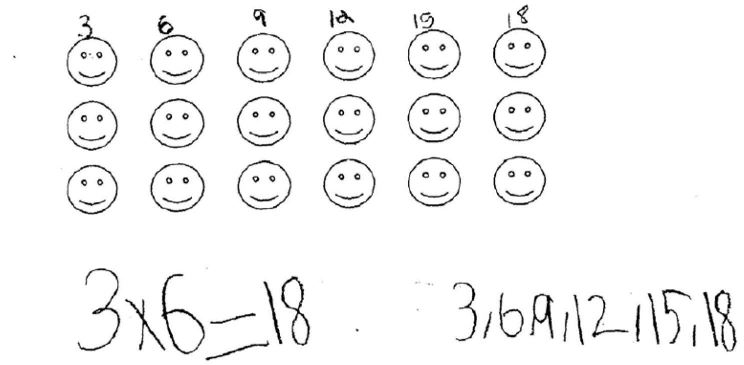
How many wheels do 29 tricycles have?



Additive Strategy

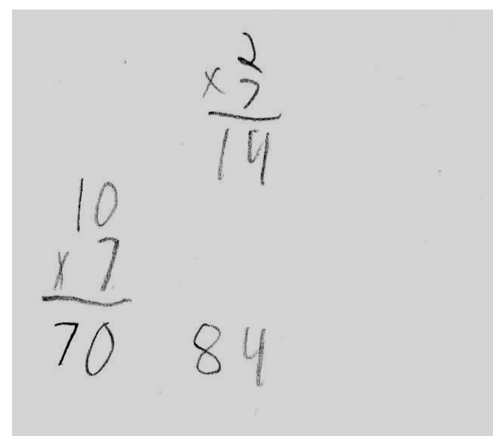
Transitional Multiplicative Strategy

Write an equation to match this picture.

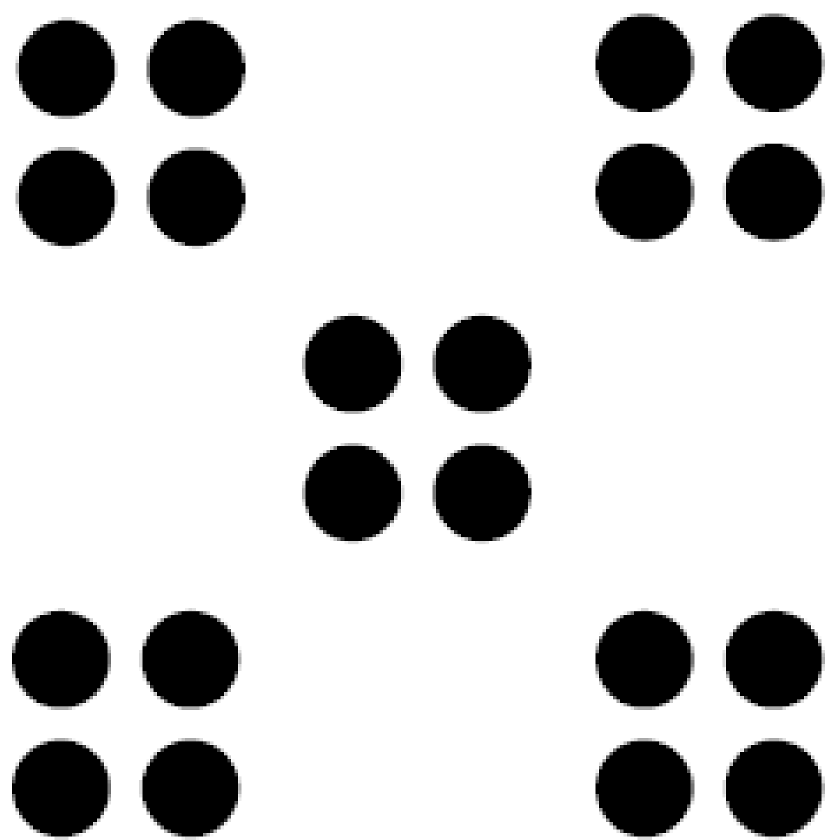


Multiplicative Strategy

Farmer Brown donated 7 dozen eggs to the senior center.
How many eggs did he donate?



"quick images" can help students move from counting by ones to multiplicative strategies



Although these are found in some elementary math programs, few teachers used them or understood the importance of them.

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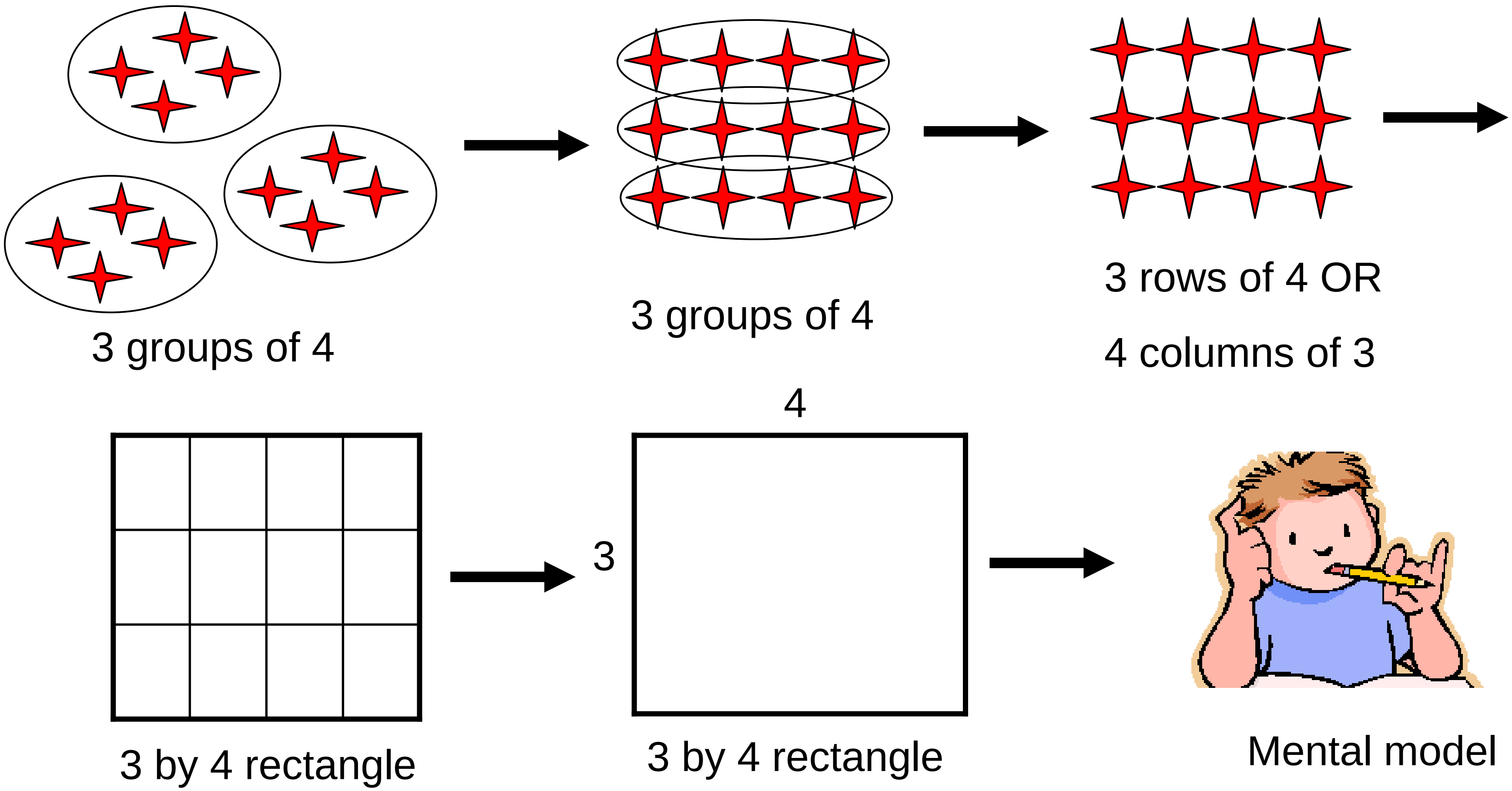
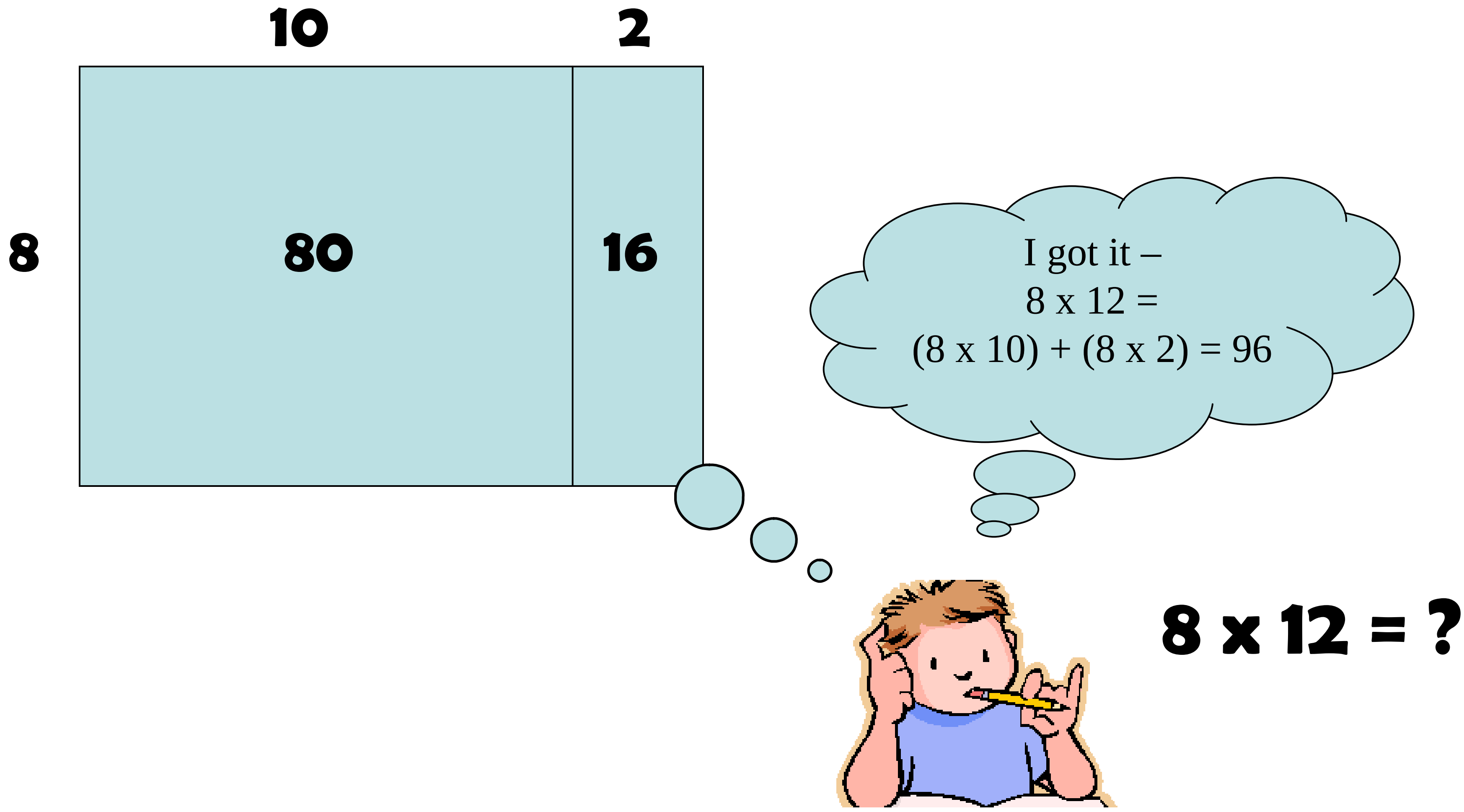
Transitioning from physical models to mental models




10
2
8
80
16
I got it –
8 x 12 =
(8 x 10) + (8 x 2) = 96
8 x 12 = ?

Back to Fractions

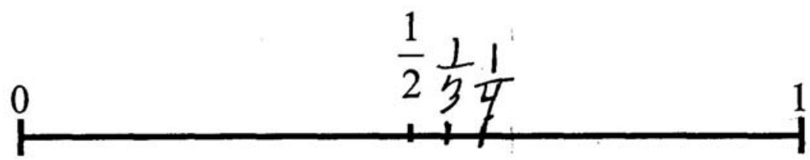
- Prevalence of inappropriate whole number reasoning
- Importance of modeling to build understanding
- Importance of developing a variety of reasoning strategies

Inappropriate whole number reasoning

According to research, some students may see a fraction as two whole numbers (e.g., ¾ as a 3 and 4) inappropriately using whole number reasoning, not reasoning with a fraction as a single quantity. (Behr, M., Post, T., Lesh, R., and Silver, E. (1983)*;* Behr, Wachsmuth and Post, (1984)*; VMP OGAP Study (2005))*

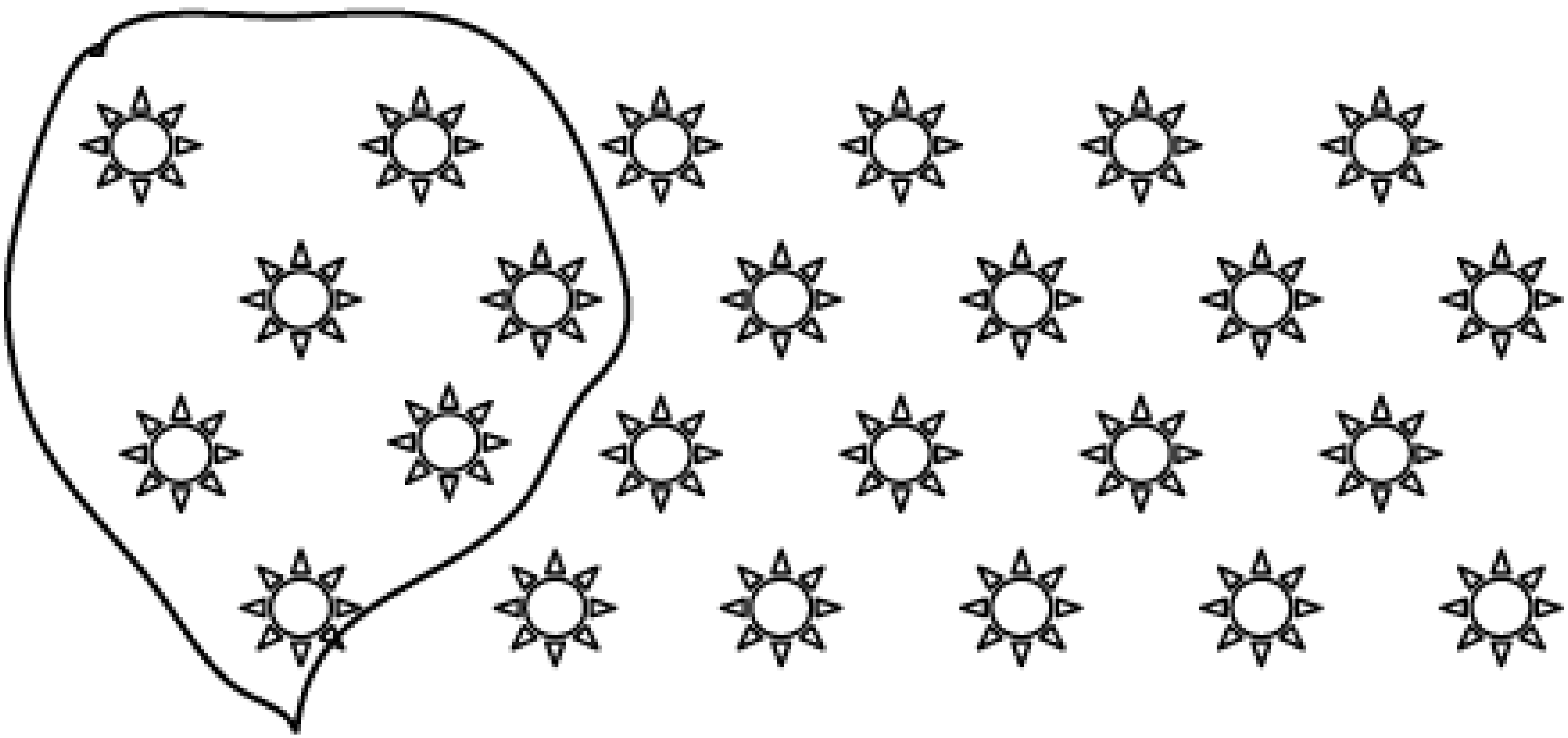
Place $\frac{1}{3}$ and $\frac{1}{4}$ in the correct location on the number line below.

Explain your answer using words or diagrams.



I chose these spots because, it says $\frac{1}{2}$, and then $\frac{1}{3}$ comes after $\frac{1}{2}$, and then $\frac{1}{4}$ after $\frac{1}{3}$ because it goes 1, 2, 3,4, and so that is how I think.

Circle 7/12 of the set of suns.



A) The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:

a) 20

b) 8

c) $\frac{1}{2}$

d) 1

Use words, pictures, or diagrams to explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20.

Modeling

- ***Models are a means to the mathematics, not the end.*** *The purpose for using multiple models with students is to provide experience that varies "features" not related to fraction ideas (e.g., shape) across different models and, therefore, facilitate the development of a generalized understanding.* (Dienes, Z. (1970))

- *… area, set, and linear models differ in the challenges that they present students.* (Hunting, R.P. (1984) cited in Bezuk, N.S., and Bieck, M., (1993)

- *…when finding the fractional part of whole it is more difficult when the number of parts in the whole is a multiple of the denominator than when the number of parts is equal to the denominator.* (Bezuk, N.S., and Bieck, M. (1993); VMP OGAP Exploratory Study (2005))

Ways to address evidence and research:

Based on the evidence of the pre-assessment data, the students in my fifth grade math class seem to be quite comfortable using the area model (using circles and rectangles), especially when they are given a "blank" whole (not partitioned) or if they create their own whole. There were evident misunderstandings when the denominator did not equal the number of partitions, or if the number of partitions was a multiple of the denominator.

Based on this evidence and on research, I plan to be sure to use the area, set and linear models interchangeably throughout the unit. In addition, it will be important to give the students lots of opportunities with area models in which the number of partitions does not equal the denominator. Because Mathland relies quite a bit on "circle" area models, I will be explicit about the limitations of the circle model (and student drawn models in general), and offer opportunities so the students can experience those limitations.

The pre-assessment information also pointed out that many of the students were confused about the definition of "the whole", and how the size of the whole impacts the fraction being considered. As I think about beginning the unit on fractions, it will be important to emphasize the fraction as a single quantity, and the importance that the whole plays when comparing and/or ordering fractions. A clarification will need to be made on the whole in set models, and strategies/experiences given to define the whole and "equal parts" (ie. Number of items in a set, regardless of size of the items).

This goes along with a misconception that the students showed in the pre-assessment of over relying on the size of the denominator to determine the size of the fraction. It will be very important to allow students to use the various models with many fractions (unit fractions, fractions with same denominators and different numerators, fractions with different numerators and denominators) throughout the unit – and see how those fractions are represented in the area, linear and set models. This will help address their understanding of the magnitude of fractions and the roles the numerators and denominators play in comparing the size of fractions.

Finally, the pre-assessment information showed that students did not often rely on benchmark fractions to compare or order fractions. It will be important to emphasize benchmark fractions throughout the unit and as a strategy for comparing determining the magnitude of other fractions.

1) What is the evidence from the pre-assessment that the teacher uses for planning?
2) What instructional decisions did the teacher make and how were they linked to the evidence or research?

Initial Planning Based on Knowledge of Research and Evidence in Pre-assessment

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Notes at the conclusion of the Unit

There were several instructional strategies that I focused on throughout the fractions unit. One such strategy was to use the number line *every day* (instead of just in the third week, as suggested by the Mathland program). Noting their limited use and exposure to number lines (previously) in the Mathland program, we had many initial discussions about how number lines work (including beginning and ending points, equal intervals and relationships of numbers). With every fractions activity we did, we placed many of the different fractions on a large number line that was on our class white board. Students were encouraged to find the point on the number line where different fractions they worked on would go. More specifically, and outlined on the "Aha" record sheet, the students were asked to place the fractions they worked on in the "Equivalent Chart" activity on the number line. This was just another model (in addition to the area model used in the activity) for students to visualize equivalent fractions and make connections between fractions and patterns in numbers.

We also spoke daily of benchmark fractions and compared "our" fraction to the different benchmarks. Since we often talk about "reasonable estimates", the students continuously made great connections, and began to think of fractions more easily as a single quantity. I do believe using the different models interchangeably, as well as at the same time continuously, made a big difference for the students. They soon became naturally able to think of fractions in different ways.

In addition, throughout the unit, students were asked to represent fractions in different ways (area, linear, set models) and present their representations to the class. The students really enjoyed this daily activity, and could question one another about the models and representations they chose to represent their fraction. The conversations shifted from the students and me, to the students and one another. They became really comfortable talking about their representations, explaining their reasoning and helping one another. I believe this made it much easier to use different reasoning strategies later in the unit when we began to compare and order fractions.

Examples of teacher interventions (response to inappropriate whole number reasoning and lack of use of models)

- Use modeling to build concepts
- Emphasis on number line
- Emphasis on relative magnitude of fractions using modeling and other reasoning strategies

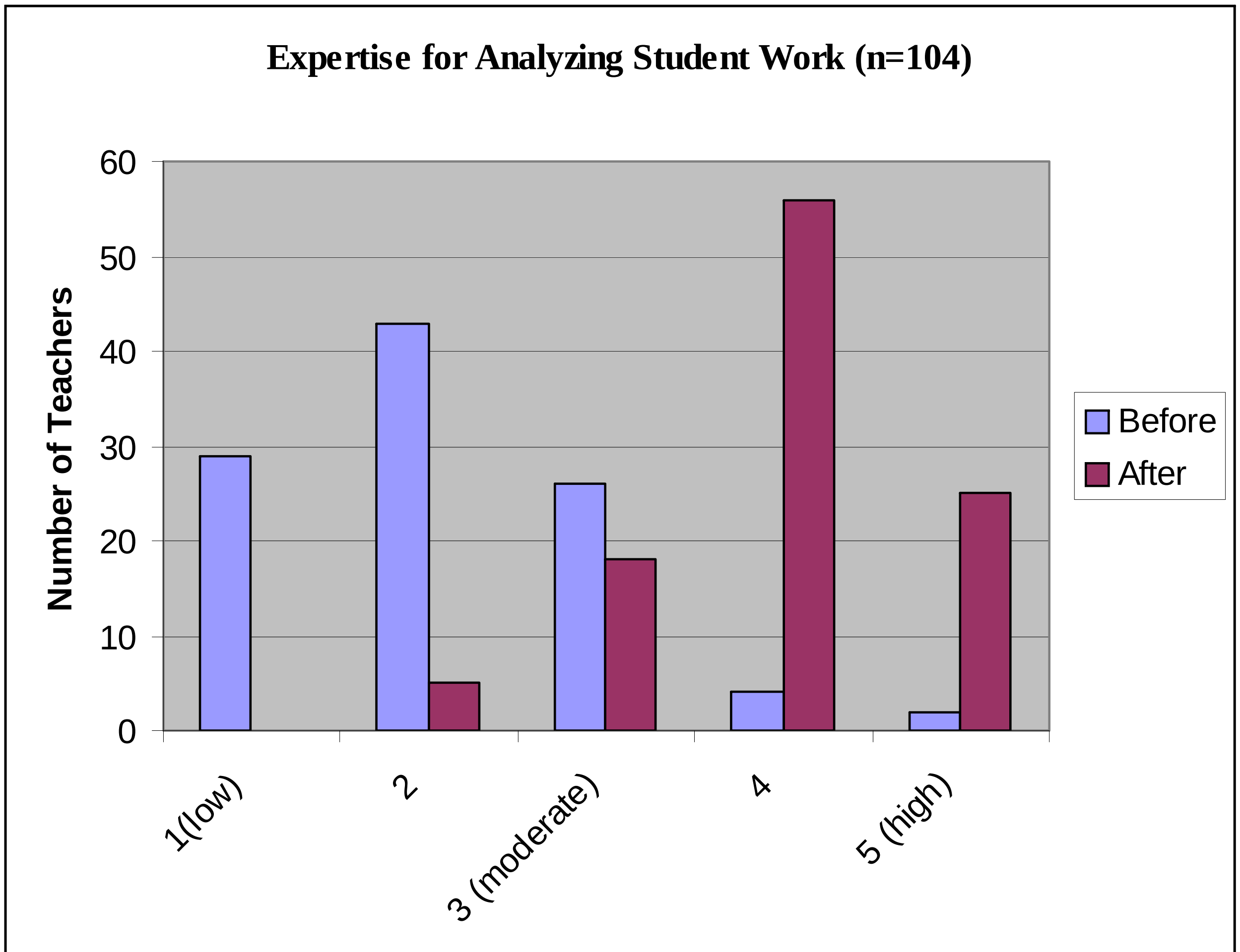
OGAP Exploratory Studies (2004, 2005) and 2006-2008 Roll-outs

What do *teacher leaders and teachers* say about their experience in relationship to the stated goals and the use of OGAP formative assessment system?

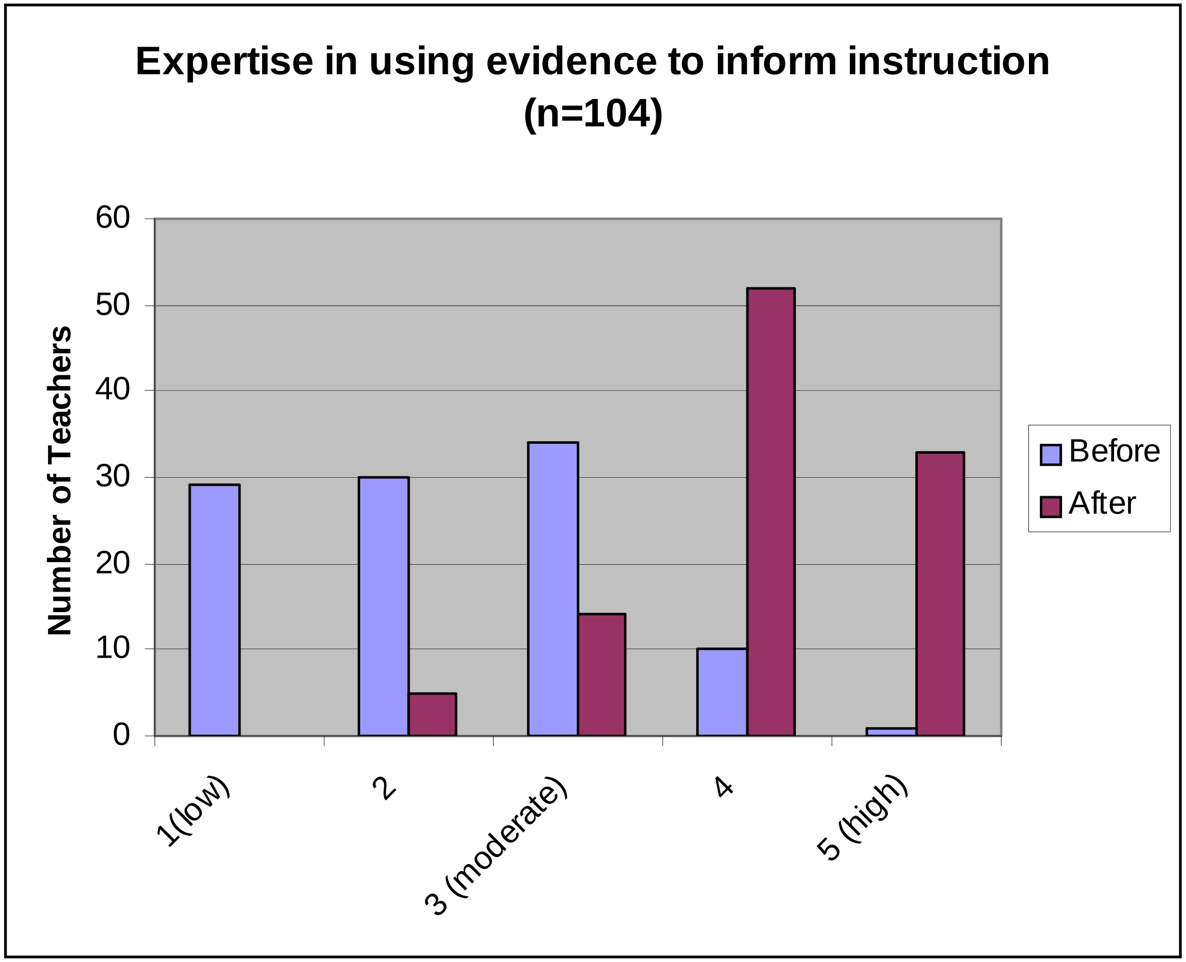
Results based on a spring 2007 online survey

Expertise for analyzing student work (for evidence of developing understanding, common errors and misconceptions)…

Before and After Experience

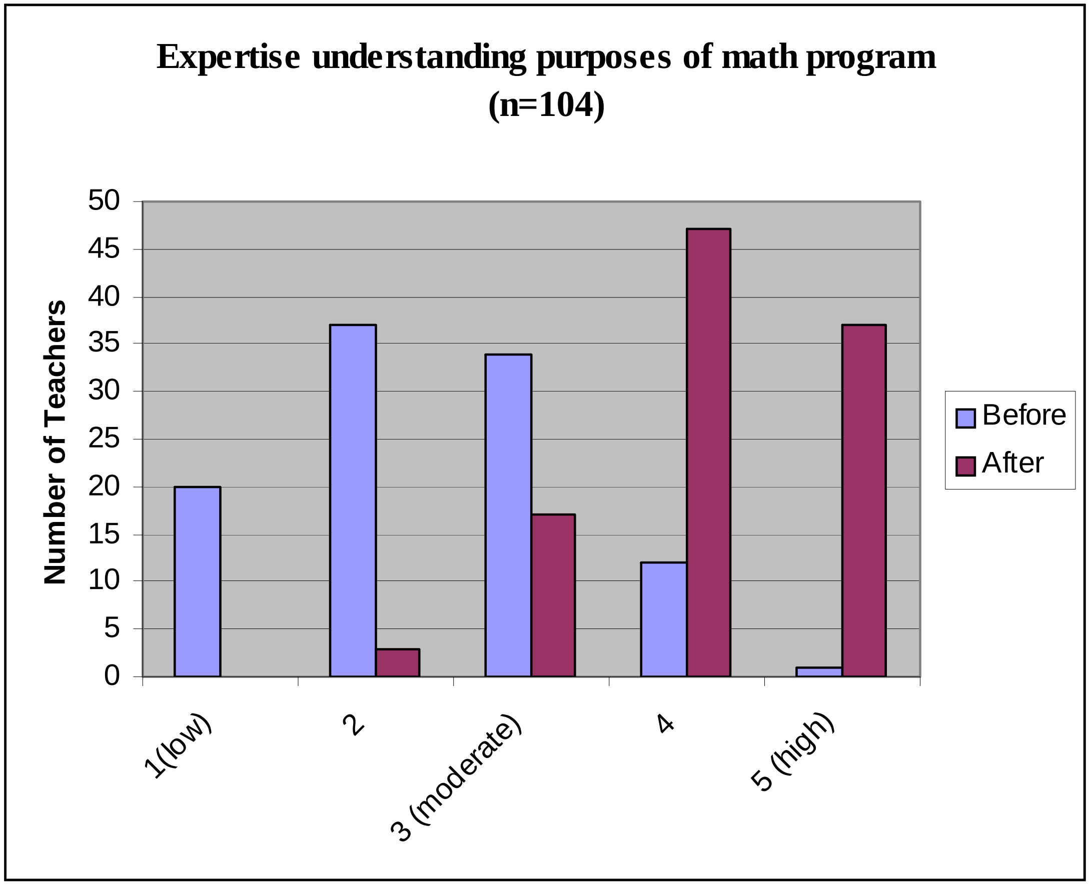


Expertise in using evidence in student work to inform instruction…



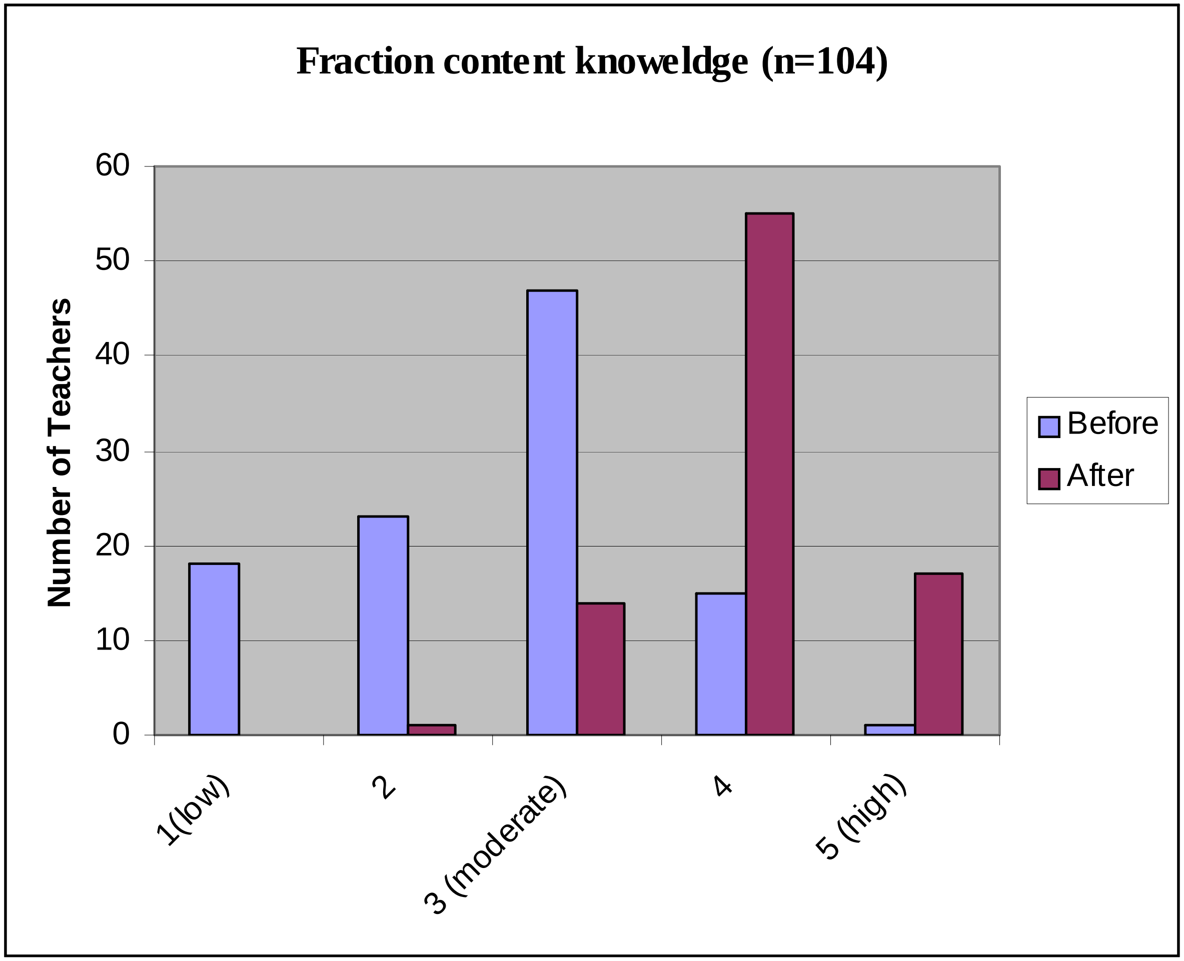
Understanding purposes of activities in mathematics program…

Before and After Experience



Fraction content knowledge…

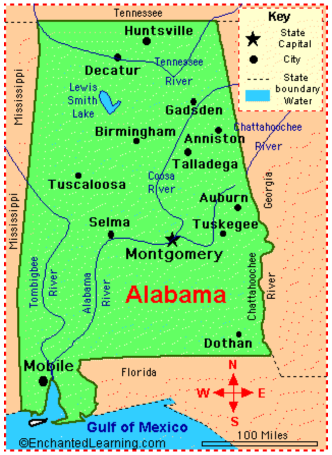
Before and After Experience



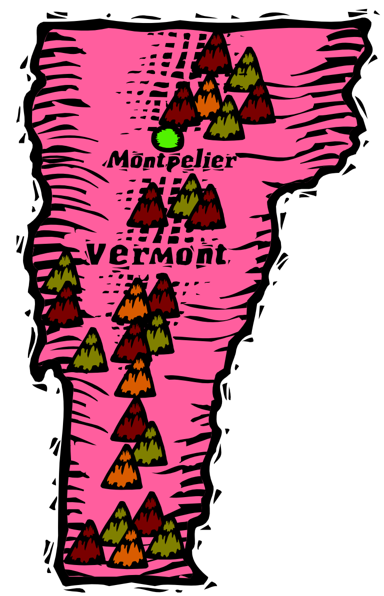
In a post survey of Vermont teacher leaders and their mentees:

95% (98/104) of respondents recommended that other teachers in their school use the OGAP fraction materials and processes.

87% (90/104) of respondents indicated that they will use OGAP multiplication and division materials when they became available.



OGAP Scale-up Model



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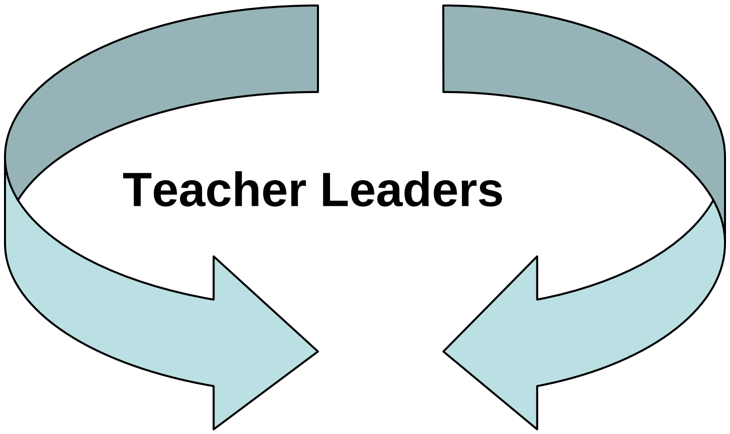
Principles for Scaling-up…

… based on findings from the 2005 Exploratory Study and recommendations of OGAP National Advisory Board

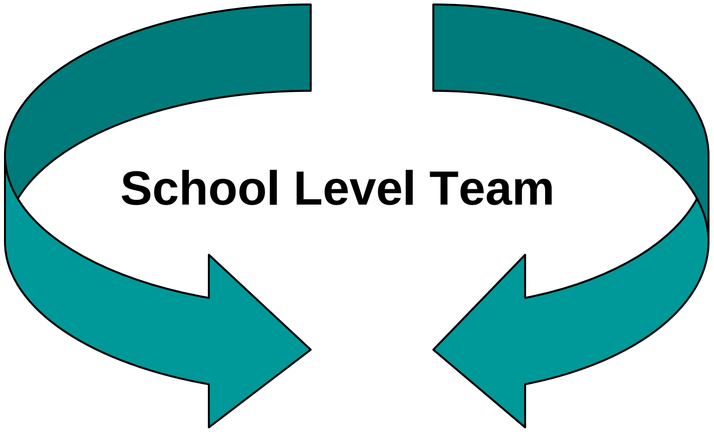
- **Capitalize on teacher leadership or school teams**
- **Provide professional development to support implementation**
- **Provide resources and support materials necessary for effective implementation.**
- **Provide mentor support during implementation**.

OGAP Fraction Scale-up – Capitalizing on Teacher Leadership



Phase I: Teacher Leader learning and experience

Phase II: Professional development support for mentees



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Vermont District OGAP Scale-up

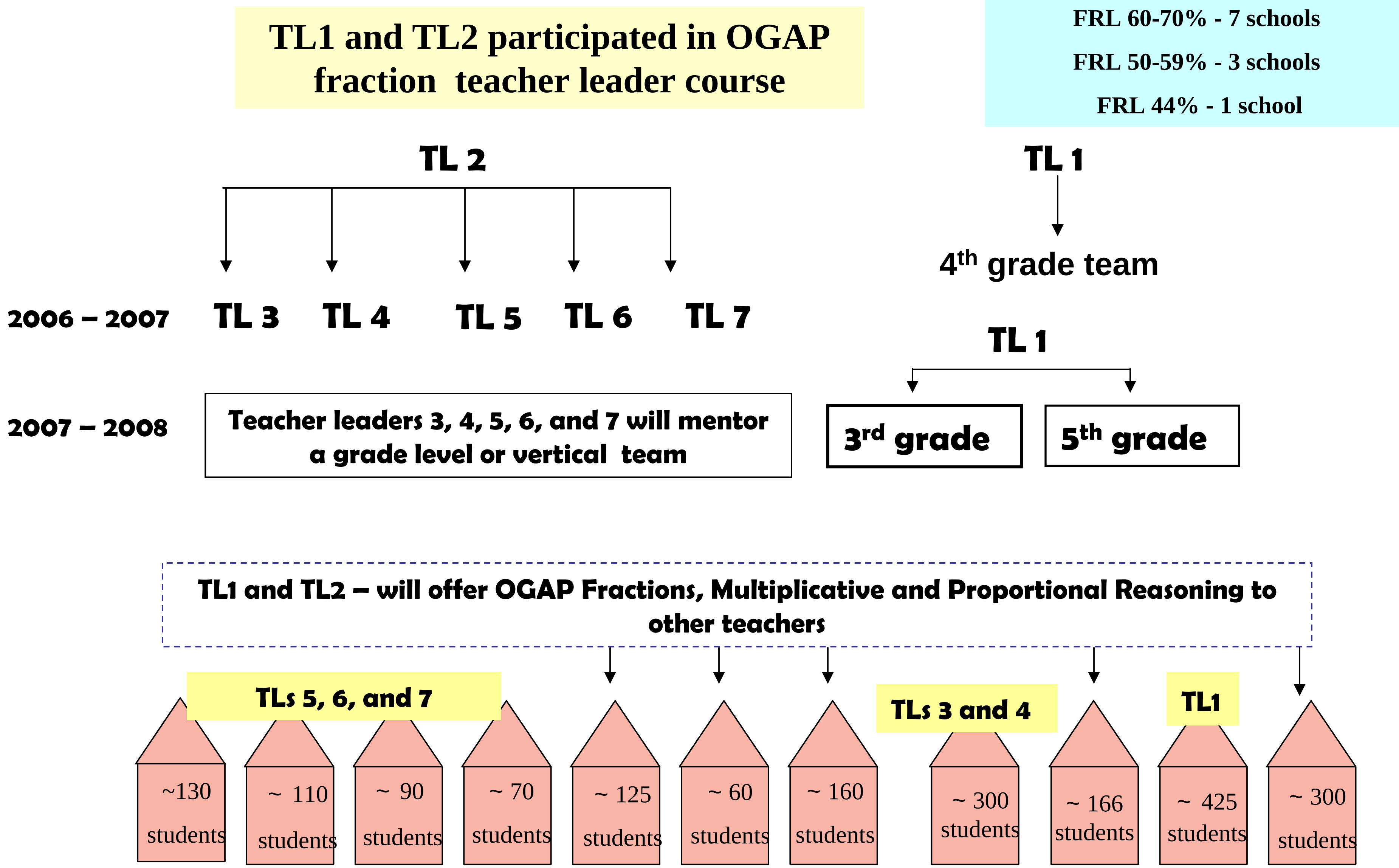
Characteristics of District

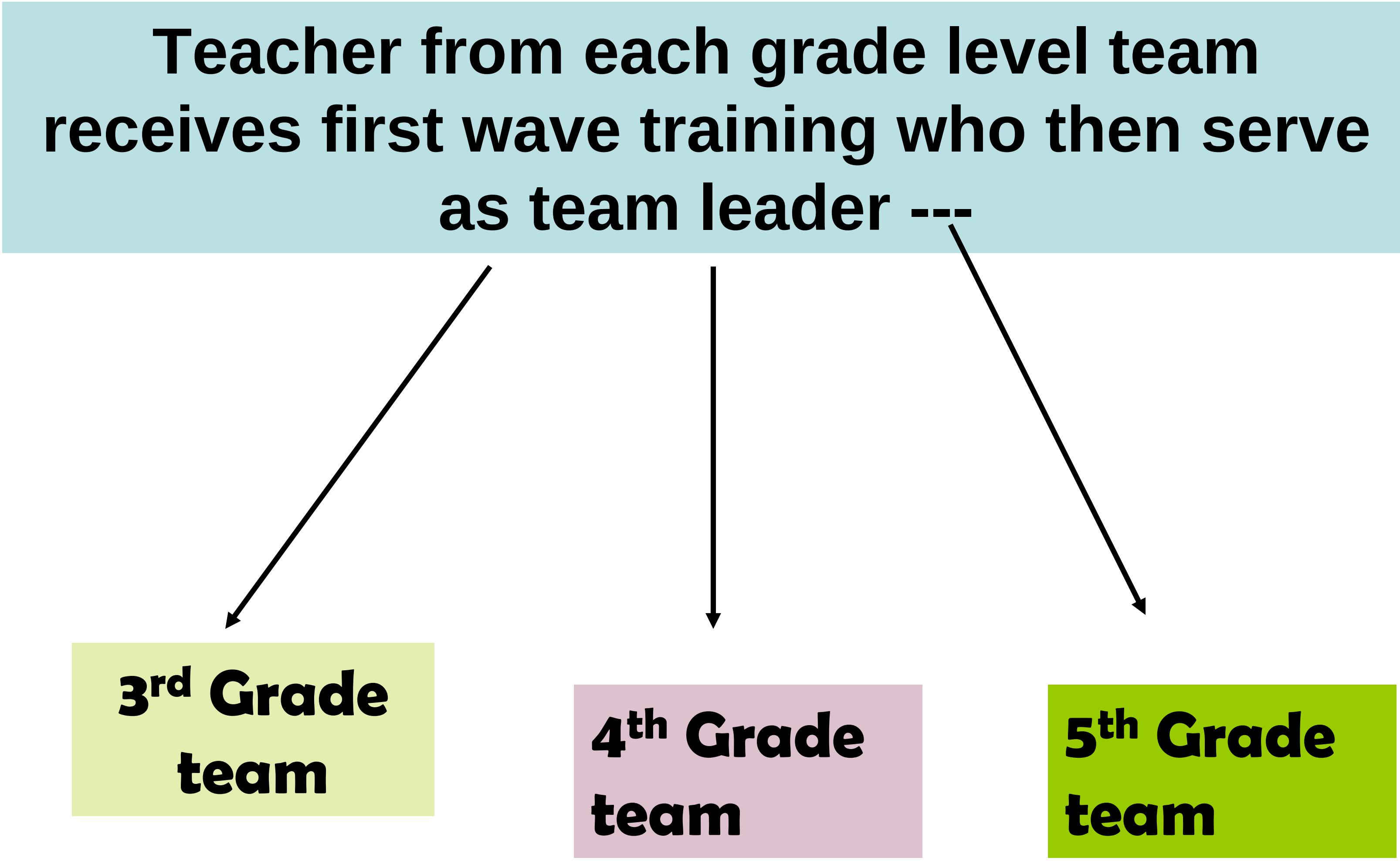
2100 students (k – 8)

FRL 60-70% - 7 schools

FRL 50-59% - 3 schools

FRL 44% - 1 school





Teacher Leadership and Teams

Grade Level Teams/School

3rd Grade team

4th Grade team

5th Grade team

School Training - everyone

What you get ---

- **Professional Development**
  - In the use of OGAP formative assessment materials and processes.
  - on the substance of the cognitive research that is foundational to the OGAP materials and processes.
  - Use of the materials “real time” with students with links to mathematics programs.

Tools and Resources

  - Item banks
  - Strategies and related tools for analyzing student work

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For more information…

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Vermont Mathematics Partnership

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&

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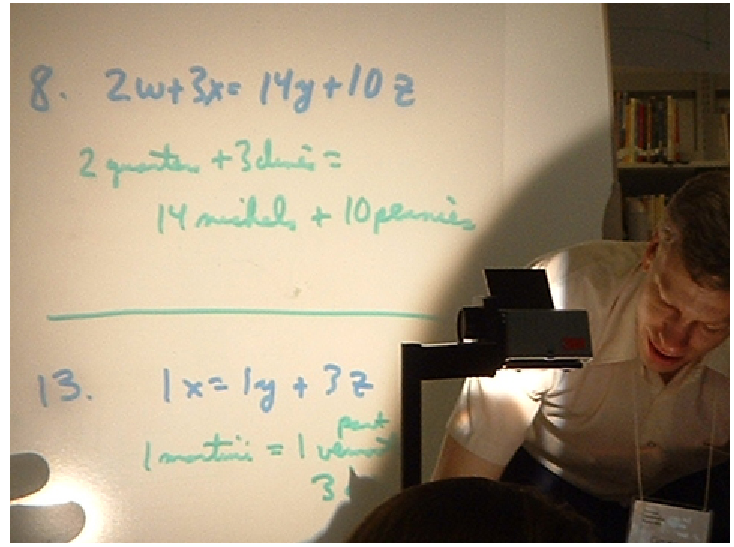

Mathematicians and Educators working together to help all Vermont children succeed in mathematics

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