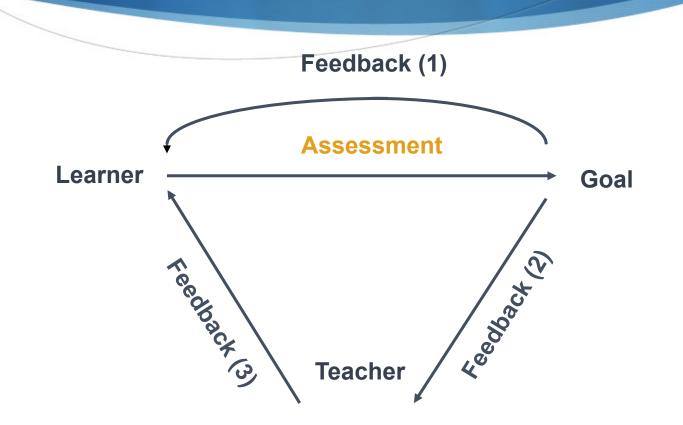
Learning Trajectories: A research lens for enhancing formative assessment.

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Take Aways

- Effective math instruction and planning for instruction requires careful attention to evidence of student thinking.
- Using learning trajectories/progressions to understand evidence of student thinking provides actionable information based on research on how students learn specific mathematics concepts
- The "essence" of formative assessment is the relentless attention to evidence of student understanding and intentional and systematic use of the evidence for planning and instruction. (Popham, 2012)

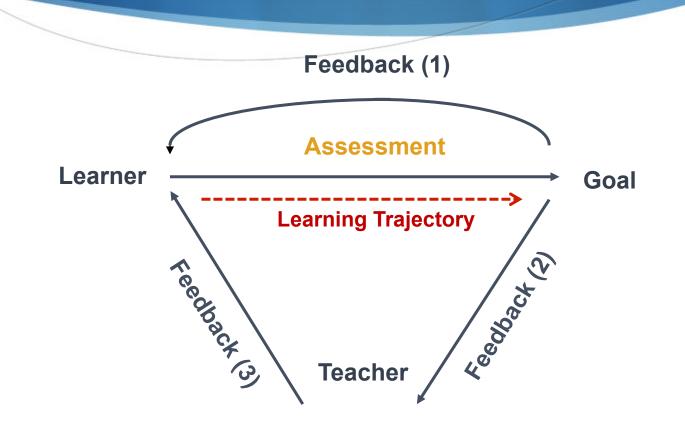
Formative Assessment

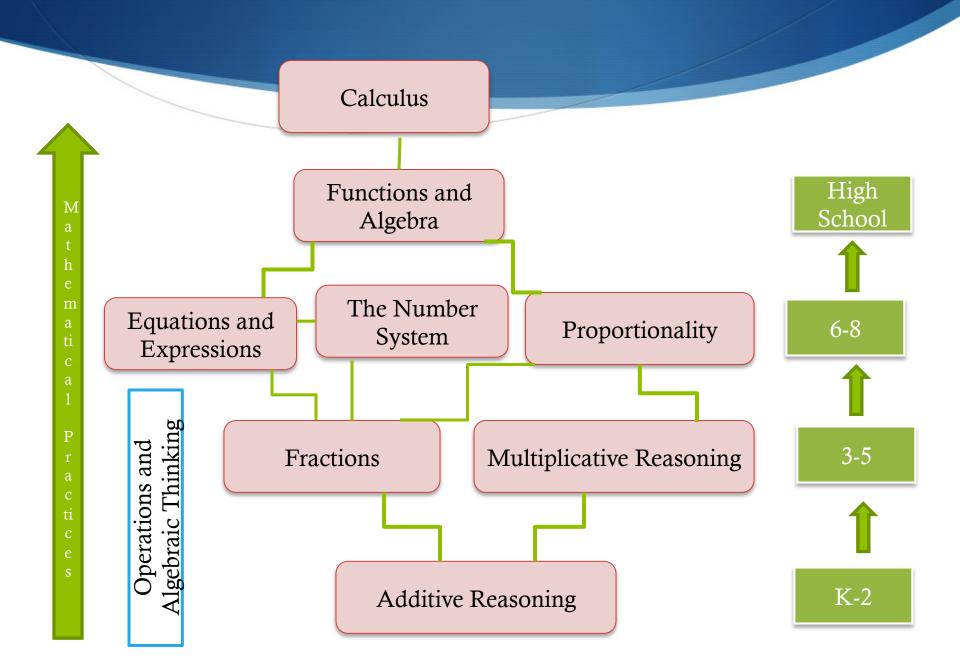


Learning Trajectories

- "Learning progressions are descriptions of the successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a topic" (NRC, 2007, p. 214)
- Developmental progressions of strategies, concepts and levels of student thinking in particular mathematical domains
- Links research on learning and instructional practice

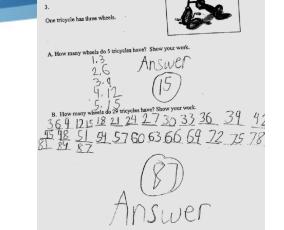
Formative Assessment





Looking at Student Work

Review the samples of student work



What do you notice? Make some observations



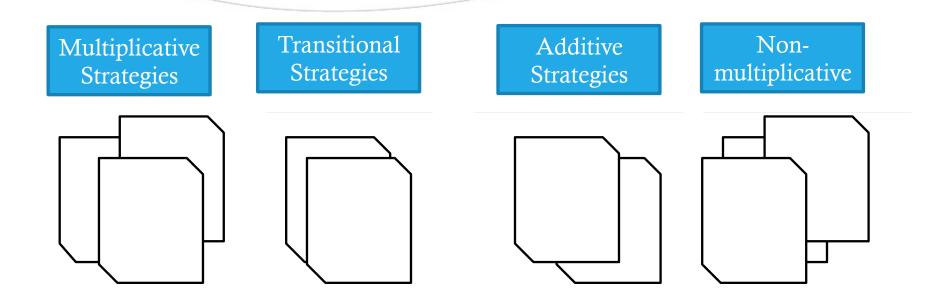
Multiplicative Reasoning Framework – Multiplication

Multiplicative Strategies Algorithms Distributive Property Associative Property Doubling & Helving Partial Products Traditional $4 \ge 16 = 4(10 + 6)$ $(8 \times 2) \times 5 = 8(2 \times 5)$ $16 \times 4 = 8 \times 8$ Aultiplicative 21 $= 8 \times 10$ = 4(10) + 4(6)= 6416 16 <u>x 42</u> 12 = 40 + 24= 80<u>x 42</u> 32 = 6420 640 672 240 Known or Derived Fact Commutative Property Powers of Ten nts may move up and down between multiplicative, transitional, additive, an situations and problem structures (Kouba & Franklin, 1995; VMP OGAP, 2006 400 $5 \times 400 = 5 \times 4 \times 10 \times 10$ Applies understanding of place value, properties, and relationshi $16 \times 4 = 4 \times 16$ $4 \times 6 = 24$ 672 Transitional Strategies Open Area Model Considers both dimensions of an array or area model Area Model 30 Considers BOTH dimensions of an 20 600 160 arra¥ or area model, (90) 45 + noving away from needing to see every 180 48 collare lini ransitional 38 x 26 = 988 $15 \times 9 = 135$ $6 \times 4 = 24$ Early Transitional Strategies Area Model - 6 x 4 = 24 Skip Counting 3, 6, 9, 12, 15 Skip Counting with Equal groups in an array a Model Building up • • • :• 3 + 3 + 3۰ • ning stu ٠ ٠ Unitizes into groups and sub-groups ng upon the strength of multiplicative reasonin; ative strategies as they interact with diffe arit pr 3 Q 12 15 6+6ĥ 12 Considers only ONE dimension Considers BOTH dimensions of an array or area model of an array or area model Additive Strategies Repeated addition with or without a model - 3 x 4=12 3+3+3+3=12Additive Subitizing in small groups Early Additive Strategies Modeling, counting by ones Modeling, counting Inconsistent Depe by subgroups Grouping \odot Ø \odot 3x4 = 12۲ 10 12 Non-Multiplicative Strategies Underlying Issues/Errors Misinterprets the Adds or subtracts factors Uses incorrect operation remainders Doesn't consider - Error in: calculation, place value, Models factors incorrectly Not enough information reasonableness of vocabulary, property or relation-Units inconsistent Guesses Uses procedures incorrectly solution ship, equation, or model or missing

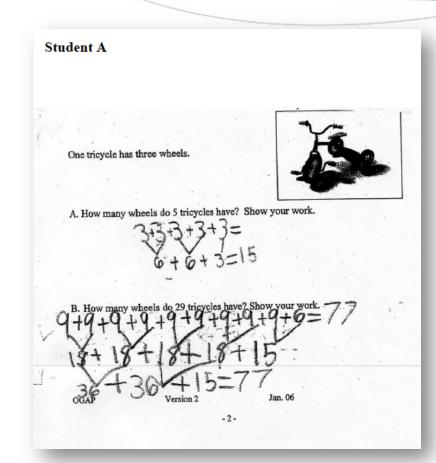
An Example: The OGAP Multiplicative Reasoning Progression

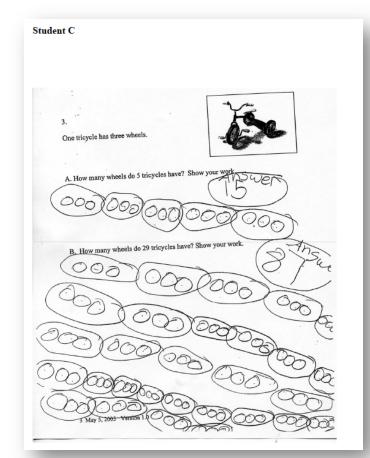
> This is a derivative product of the Vermont Mathematics Pertnership Ongoing Assessment Project (OGAP) which was funded by NSF (EHR-0227057) and the US DOE (S3664020002). © 2012 Marge Petit Consulting, MPC, E. Hulbert, R. Laird. Version 27 January 2013.

The OGAP Sort



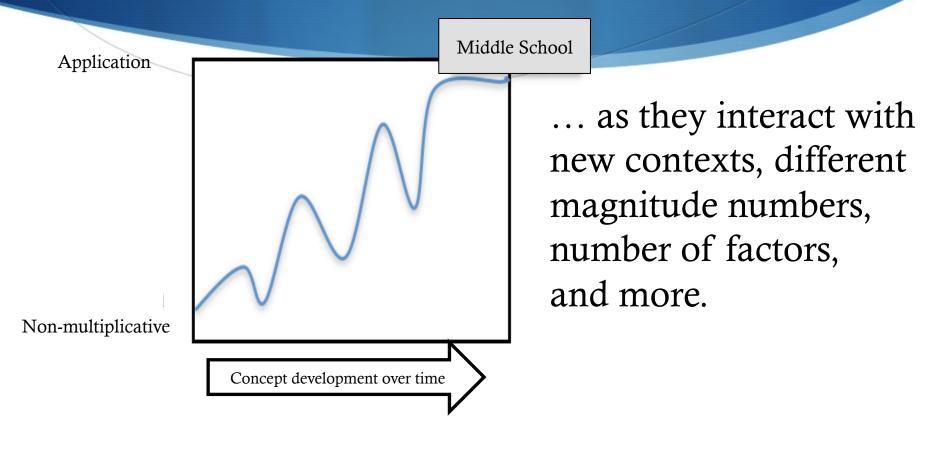
Instructional Implications





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Students move back and forth...

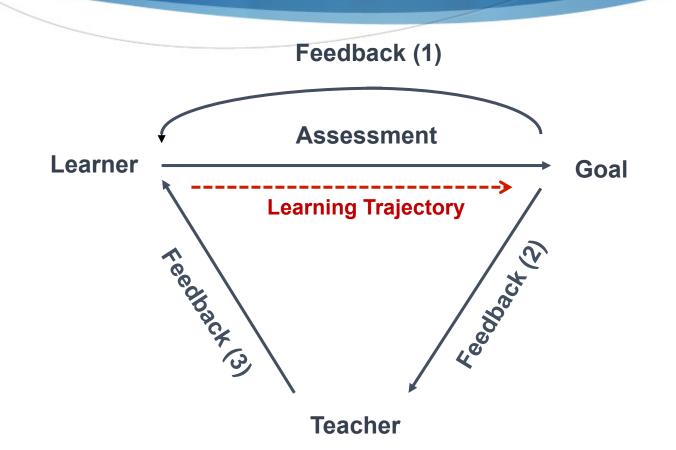


Petit, Laird, Marsden, & Ebby, in press 2015

CCSSM

Grade	CCSSM Multiplicative Problem Situations (BOLD = new for grade level)	CCSSM Multiplicative Strategies
2	Equal groups	Repeated addition with an array
3	Equal groups, arrays, equal measures, beginning area	Solve multiplication and division problems using strategies based on place value and properties of operations.
	Multiplication and division within 100 and 1 digit x multiples of 10 (e.g., 5 x 50)	
4	Equal groups, equal measures, multiplicative comparisons, measurement conversions within systems, area Multiply 1 digit x up to 4 digits, and 2 digits x 2 digits. Divide up to 4 digits by 1 digit numbers.	Solve multiplication and division problems using strategies based on place value (e.g., partial products, area models) and the properties of operations and relationships (e.g., commutative, associative, and distributive, inverse relationship between multiplication and division).
5	Equal groups, equal measures, multiplicative comparisons, measurement conversions between systems, area, scaling (multiplicative change) Fluently multiply whole numbers. Divide up to 4 digits by 2 digits.	Solve multiplication problems using efficient strategies (e.g., partial products, traditional algorithms). Solve division problems using strategies based on place value (e.g., partial quotients, menus, area models) and the properties of operations and relationships (e.g., commutative, associative, distributive, and the relationship between multiplication and division).

Learning Trajectory-Oriented Formative Assessment





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- Popham, W. J. (2012). Forward. In E. Wylie, A. Gullickson, K. Cummings, L. Noakes, K. Norman, & S. Veeder (Eds.), Improving formative assessment to empower student learning (pp. ix-xii). Thousand Oaks, CA: Corwin Press.
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