Learning Trajectories: A research lens for enhancing formative assessment.

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Take Aways

- Effective math instruction and planning for instruction requires careful attention to evidence of student thinking.

- Using learning trajectories/progressions to understand evidence of student thinking provides actionable information based on research on how students learn specific mathematics concepts.

- The “essence” of formative assessment is the relentless attention to evidence of student understanding and intentional and systematic use of the evidence for planning and instruction. (Popham, 2012)
Formative Assessment

Learner → Assessment → Goal

Feedback (1)

Feedback (2)

Feedback (3)

Teacher
“Learning progressions are descriptions of the successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a topic” (NRC, 2007, p. 214)

Developmental progressions of strategies, concepts and levels of student thinking in particular mathematical domains

Links research on learning and instructional practice
Formative Assessment

Learner -> Assessment -> Feedback (1) -> Goal

Teacher -> Feedback (2) -> Learning Trajectory -> Feedback (3)
Looking at Student Work

- Review the samples of student work
- What do you notice? Make some observations
An Example: The OGAP Multiplicative Reasoning Progression
The OGAP Sort

- Multiplicative Strategies
- Transitional Strategies
- Additive Strategies
- Non-multiplicative Strategies
Instructional Implications

Student A

One tricycle has three wheels.

A. How many wheels do 5 tricycles have? Show your work.

\[ \begin{align*}
3 + 3 + 3 + 3 + 3 &= \frac{3 \times 5}{5} = 15 \\
6 + 6 + 3 &= 15
\end{align*} \]

B. How many wheels do 29 tricycles have? Show your work.

\[ \begin{align*}
9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 &= 81 \\
13 + 18 + 18 + 18 + 15 &= 77 \\
36 + 36 + 36 + 15 &= 77
\end{align*} \]

Student C

3. One tricycle has three wheels.

A. How many wheels do 5 tricycles have? Show your work.

\[ \text{Answer: } 15 \]

B. How many wheels do 29 tricycles have? Show your work.

\[ \text{Answer: } 77 \]
Students move back and forth... as they interact with new contexts, different magnitude numbers, number of factors, and more.

Petit, Laird, Marsden, & Ebby, in press 2015
<table>
<thead>
<tr>
<th>Grade</th>
<th>CCSSM Multiplicative Problem Situations (BOLD = new for grade level)</th>
<th>CCSSM Multiplicative Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Equal groups</td>
<td>Repeated addition with an array</td>
</tr>
<tr>
<td>3</td>
<td>Equal groups, arrays, equal measures, beginning area</td>
<td>Solve multiplication and division problems using strategies based on place value and properties of operations.</td>
</tr>
<tr>
<td></td>
<td>Multiplication and division within 100 and 1 digit x multiples of 10 (e.g., 5 x 50)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Equal groups, equal measures, multiplicative comparisons, measurement conversions within systems, area</td>
<td>Solve multiplication and division problems using strategies based on place value (e.g., partial products, area models) and the properties of operations and relationships (e.g., commutative, associative, and distributive, inverse relationship between multiplication and division).</td>
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<tr>
<td></td>
<td>Multiply 1 digit x up to 4 digits, and 2 digits x 2 digits. Divide up to 4 digits by 1 digit numbers.</td>
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<tr>
<td>5</td>
<td>Equal groups, equal measures, multiplicative comparisons, measurement conversions between systems, area, scaling (multiplicative change)</td>
<td>Solve multiplication problems using efficient strategies (e.g., partial products, traditional algorithms).</td>
</tr>
<tr>
<td></td>
<td>Fluently multiply whole numbers. Divide up to 4 digits by 2 digits.</td>
<td>Solve division problems using strategies based on place value (e.g., partial quotients, menus, area models) and the properties of operations and relationships (e.g., commutative, associative, distributive, and the relationship between multiplication and division).</td>
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Learning Trajectory-Oriented Formative Assessment

