Classroom Perspective

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FRIDAY INSTITUTE FOR EDUCATIONAL INNOVATION

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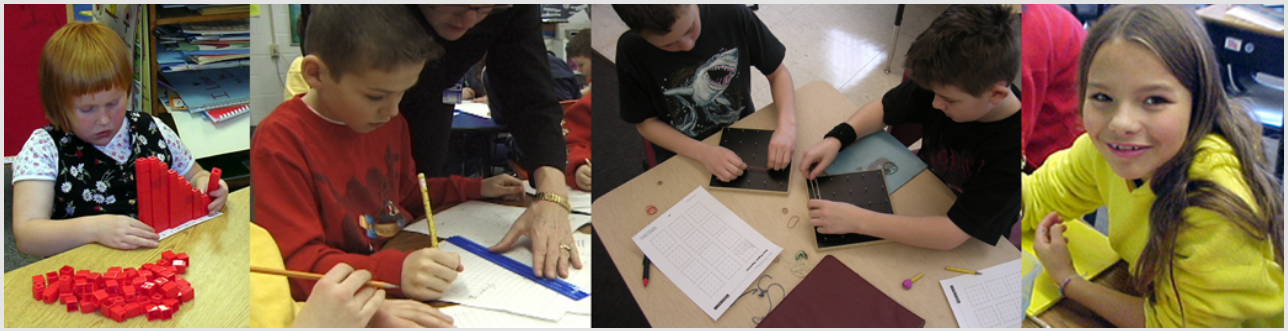


Take Aways!

- **Teacher knowledge** about the research/learning progressions is fundamental – this involves a real commitment to PD, NOT just creating tools and materials, but substantive professional development

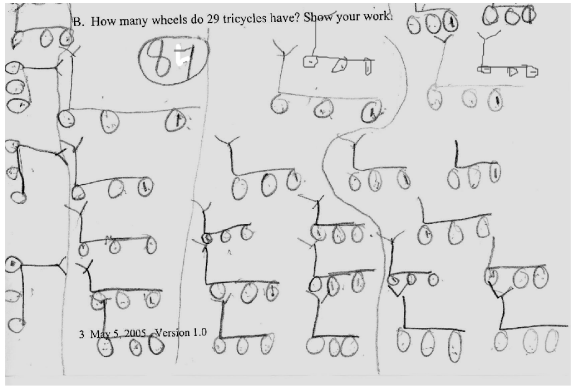
- **Tools** should:
  a) reflect the bidirectional nature of learning progressions
  b) Reflect developing understanding, common errors, and preconceptions or misconceptions that may interfere with learning new concepts r solving related problems
  c) Be at a grain size that is manageable and useable by both teachers and students

- **Student self-assessment is key**

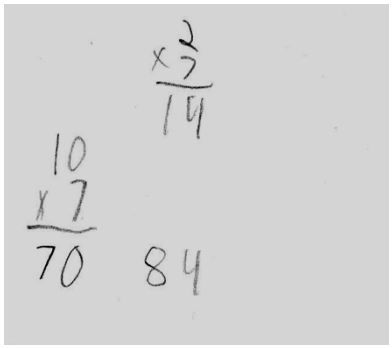
Teachers are ready!

One tricycle has three wheels.

How many wheels do 29 tricycles have?



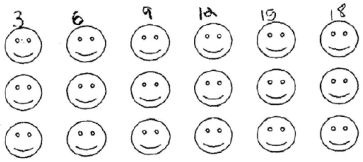
Farmer Brown donated 7 dozen eggs to the senior center.
How many eggs did he donate?



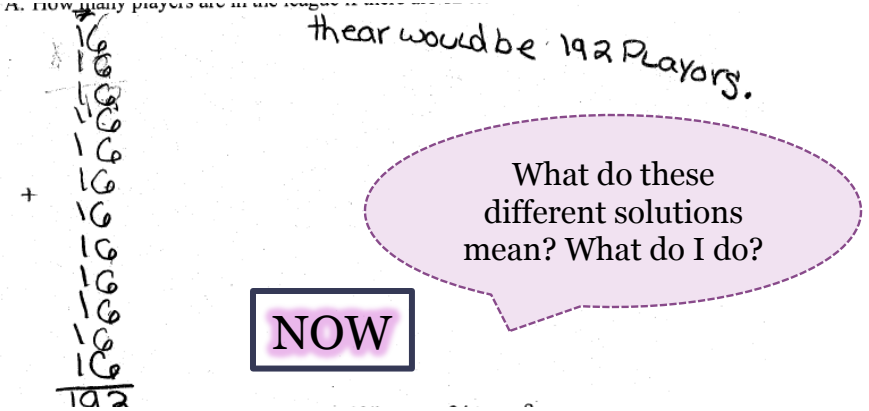
1990-2000s

Wow – look at all the ways students can solve the problem

Write an equation to match this picture. How many smiley faces are there?



There are 16 players on a team in the Smithville Soccer League. How many players are in the league if there are 12 teams?



What do these different solutions mean? What do I do?

NOW

Source of Insights 2003 - 2010

- Interaction with over 1000 educators in Vermont, Michigan, Alabama, Amman, Jordan, Nebraska, and Ohio a with OGAP materials and resources
- 3 exploratory studies using the iterative process of Design Research
- Analysis of teacher logs, interviews, surveys, student work archives, and action research projects, other.

- Fractions
- Multiplicative Reasoning
- Proportionality

The Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) formative assessment system

OGAP Principles

- ***Principle # 1: Build on pre-existing knowledge*** (*How People Learn (2000)* National Research Council)

- ***Principle # 2: Learn (and assess) for Understanding*** (Adding it Up! (*2001) National Research Council*

- ***Principle # 3: Use Frequent Formative Assessment (****Inside the Black Box, (2001) Black, P, and Wiliam, D.)*

- ***Principle # 4: Build Assessments on Mathematics Education Research*** *(Knowing What Students Know (2001) National Research Council)*

Based on these principles

- Build item banks with hundreds of items sensitive to the research
- Developed tools for collecting and analyzing evidence in student work
- Provided PD in use of tools and strategies

We initially thought it was about providing research sensitive tools – but it was really about the knowledge of research..

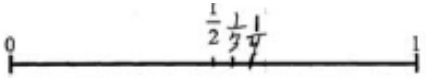
Teachers told us that knowledge of the research helped them: (OGAP, 2005)

a) better understand evidence in student work;
b) Better understand the purposes of activities in math program;
c) Make informed instructional decisions;
d) Improve first wave instruction.

Teachers told us – PLEASE tell us more on the research.

According to research, some students may see a fraction as two whole numbers (e.g., ¾ as a 3 and 4) inappropriately using whole number reasoning, not reasoning with a fraction as a single quantity.

(Behr, M., Post, T., Lesh, R., and Silver, E. (1983); Behr, Wachsmuth and Post, (1984); *VMP OGAP Study (2005))*



I chose these spots because, it says $\frac{1}{2}$, and then $\frac{1}{3}$ comes after $\frac{1}{2}$, and then $\frac{1}{4}$ after $\frac{1}{3}$ because it goes 1, 2, 3, 4, and so that is how I think.

Impact on first wave instruction!

A) The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:

a) 20

b) 8

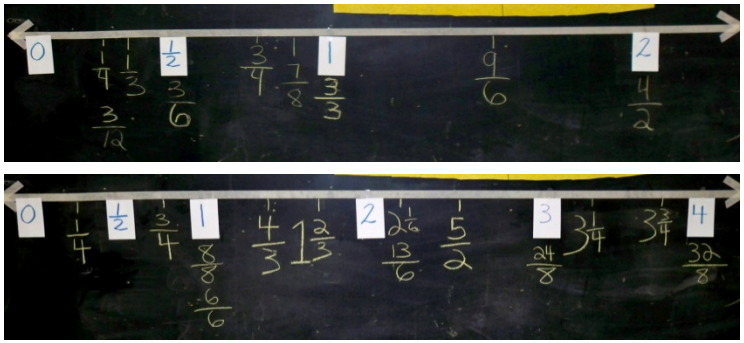
c) $\frac{1}{2}$

d) 1

Use words, pictures, or diagrams to explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to 20.

Teacher actions: A greater emphasis on magnitude and the use of number lines AS first wave instruction!



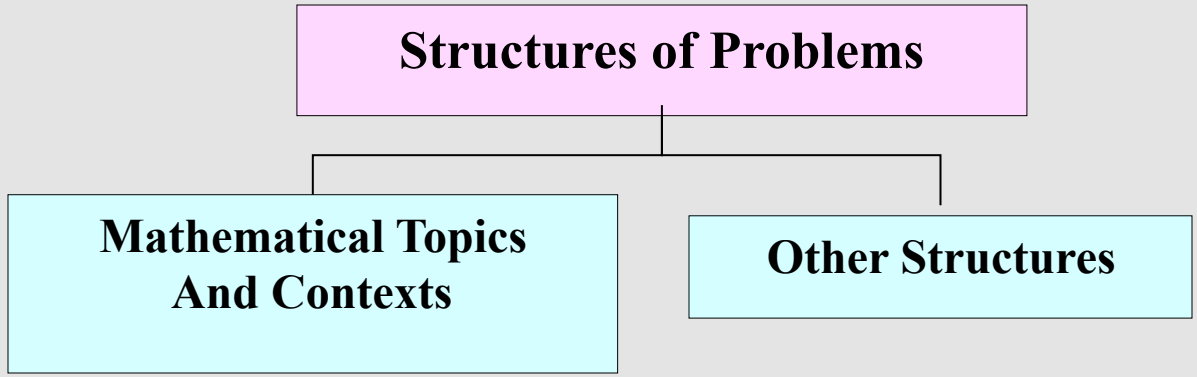
Tools:

a) reflect the bidirectional nature of learning progressions

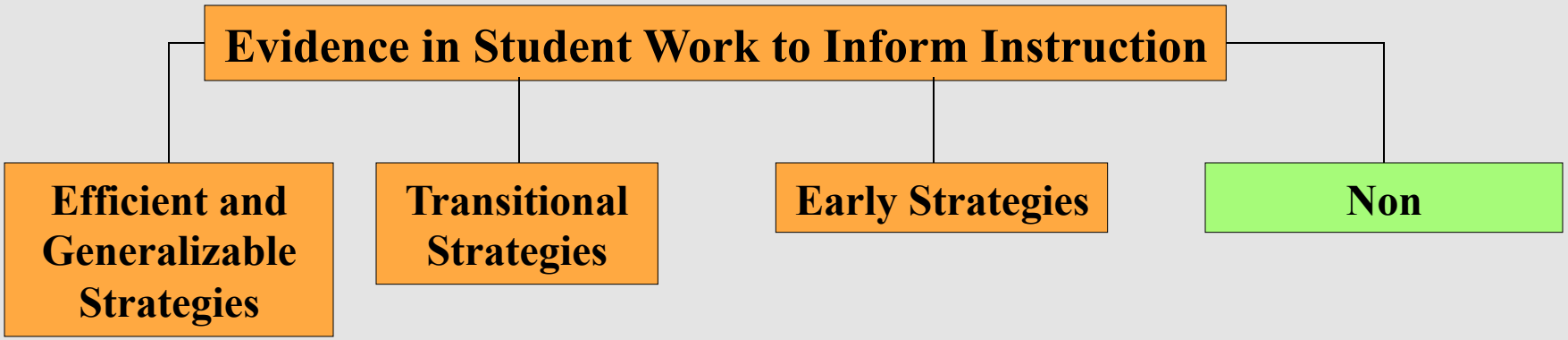
b) Reflect developing understanding, common errors, and preconceptions or misconceptions that may interfere with learning new concepts r solving related problems

c) Be at a grain size that is manageable and useable by both teachers and students

OGAP Frameworks/Progressions



Depending upon the strength of multiplicative reasoning students may move back and forth between multiplicative, transitional, additive and non-multiplicative strategies as they interact with different problem structures (e.g., context, magnitude of factors, divisors, or dividends) Kouba, V. & Franklin, K., 1995; VMP OGAP, 2006)



Created by the Vermont Mathematics Partnership funded by The National Science Foundation (Award Number EHR-0227057) and the US Department of Education (S366A020002)) October 2008

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3.

One tricycle has three wheels.

A. How many wheels do 5 tricycles have? Show your work.

15 $3 \times 5 = 15$ $5 \times 3 = 15$

B. How many wheels do 29 tricycles have? Show your work.

87

$29 \times 3 = 87$

2 29 29 29

8 7

September 2009 DRAFT

OGAP Multiplicative Reasoning Framework - Multiplication

Multiplicative Strategies

Algorithms

Partial Products: $16 \times 4 = 40 + 24 = 64$

Traditional

Distributive property: $16 \times 4 = 4(10 + 6) = 4(10) + 4(6) = 40 + 24 = 64$

Doubling and Halving: $16 \times 4 = 8 \times 8 = 64$

Associative property: $8 \times 2 \times 4 = 2 \times 8 \times 4$

Commutative property: $16 \times 4 = 4 \times 16$

Known or derived fact: $4 \times 6 = 24$

Transitional Multiplicative Strategies

Open Area Model: $16 \times 21 = 336$ (10 + 6; 20 + 1; 200, 120, 10, 6)

Area Model: $6 \times 4 = 24$ (4, 2; 16, 8)

Representing 4 x 6

Skip Counting 3, 6, 9, 12, 15

Equal groups in an array

Area Model

Equal groups

Building up: 3 + 3 + 3 + 3 = 6 + 6 = 12

(Unitizing into groups and subgroups)

Additive strategies

Repeated addition with or without a model: $3 + 3 + 3 + 3 + 3 = 15$

(Subitizing small groups)

Modeling - Counting by ones

Modeling - Counting by subsets

Depending upon the strength of multiplicative reasoning students may move back and forth between multiplicative, transitional, and additive strategies as they interact with different problem structures (e.g., context, magnitude of factors, number of factors). (Kouba, V. & Franklin, K., 1995; VMP OGAP, 2006).

Non-multiplicative Strategies

- Adds or subtracts factors
- Models factors incorrectly
- Uses incorrect operation
- Not enough information
- Guesses

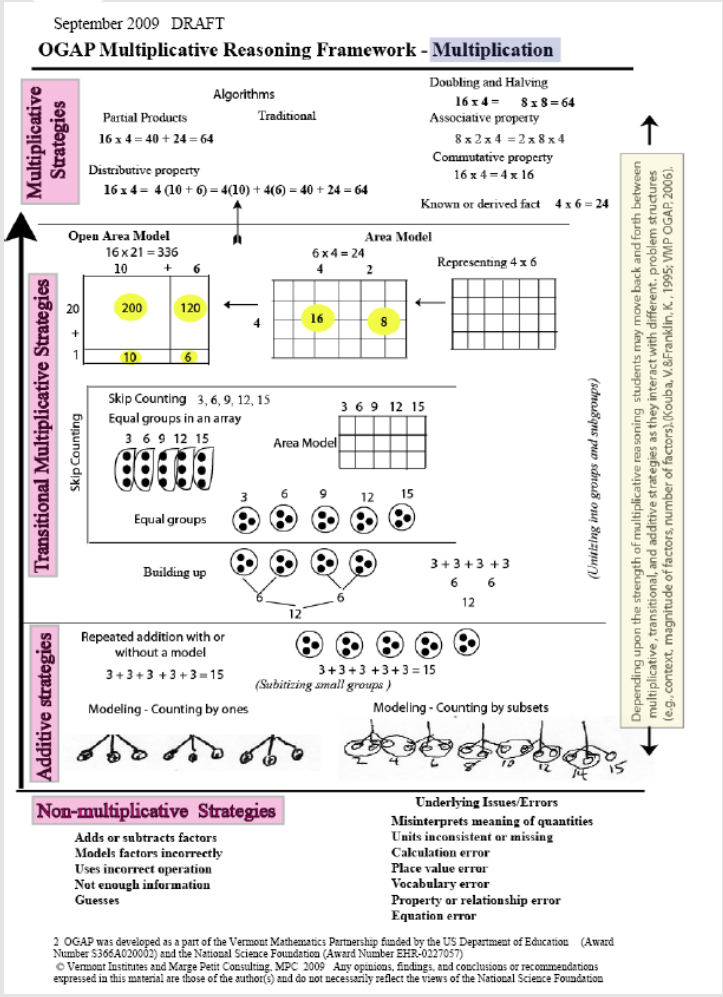
Underlying Issues/Errors

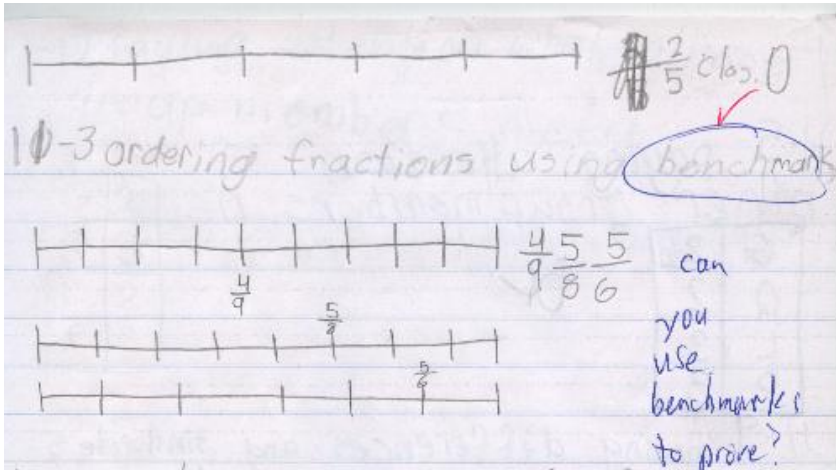
- Misinterprets meaning of quantities
- Units inconsistent or missing
- Calculation error
- Place value error
- Vocabulary error
- Property or relationship error
- Equation error

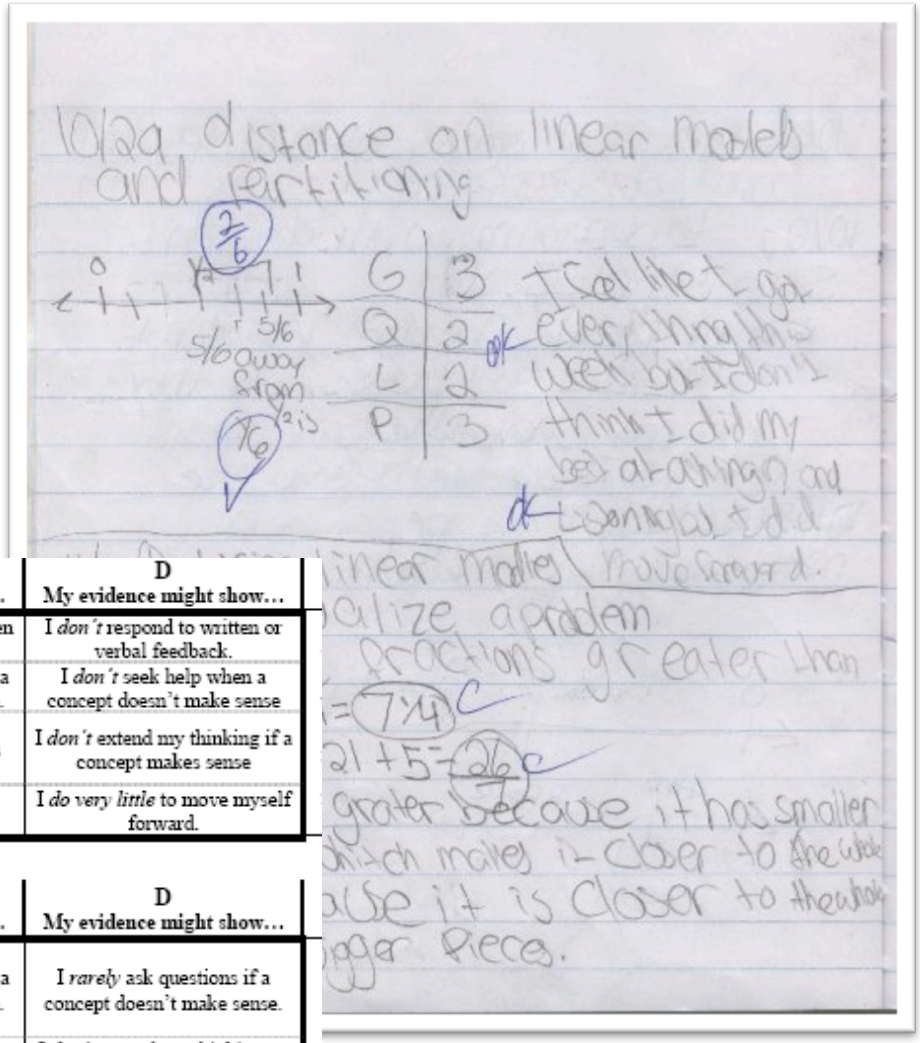
2 OGAP was developed as a part of the Vermont Mathematics Partnership funded by the US Department of Education (Award Number S366A020002) and the National Science Foundation (Award Number EHR-0227057)

Student Self-assessment

Where am I?
Where am I going?







Growth/ Progress

A My evidence might show…	B My evidence might show…	C My evidence might show…	D My evidence might show…
I respond to written or verbal feedback *consistently*.	I respond to written or verbal feedback *often*.	I *sometimes* respond to written or verbal feedback.	I *don't* respond to written or verbal feedback.
I seek help *until a concept makes sense to me*.	I seek help *when* a concept doesn't make sense.	I *sometimes* seek help when a concept doesn't make sense.	I *don't* seek help when a concept doesn't make sense
I *actively* extend my thinking if a concept makes sense.	I extend my thinking if a concept makes sense.	I *sometimes* extend my thinking if a concept makes sense.	I *don't* extend my thinking if a concept makes sense
I *actively* move *myself* forward throughout the week.	I move myself forward throughout the week.	*Sometimes* I move *myself* forward.	I *do very little* to move myself forward.

Questioning

A My evidence might show…	B My evidence might show…	C My evidence might show…	D My evidence might show…
I *consistently* ask *specific* questions if a concept doesn't make sense, and I work toward math understanding.	I *often* ask questions if a concept doesn't make sense.	I *sometimes* ask questions if a concept doesn't make sense.	I *rarely* ask questions if a concept doesn't make sense.
I *actively* try to *extend* my thinking on a problem.	I *often* extend my thinking on a problem.	I *sometimes* extend my thinking on a problem.	I *don't* extend my thinking on a problem.
I *actively seek* challenges if a concept is solid.	I *take on* challenges *given to* me if a concept is solid.	I *occasionally attempt* a challenge given to me.	I *don't* take on challenges.
I *actively seek* the answers to questions I'm asking/ thinking.	I *often* seek the answers to questions I'm asking/ thinking.	I *sometimes* seek the answers to questions I'm thinking/ asking.	I *rarely* seek the answers to questions I'm thinking/ asking.

Draft Rubric – Eric Eeley, Crosset Brook Middle School, Waterbury, VT.

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Vermont Mathematics Partnership Ongoing Assessment Project. Exploratory Study, student work samples, 2005, 2006, 2007.
Student work samples used with permission of the Vermont Mathematics Partnership funded by the US Department of Education (Award Number S366A020002) and the National Science Foundation (Award Number EHR-0227057)

For more information about OGAP go to www.margepetit.com