OGAP

Engineering Questions and Problem Structures

About this Page: The problems below illustrate how OGAP items are engineered to elicit specific evidence in relationship to both a mathematical goal and in response to math education research by intentionally altering problem structures. (These examples do not represent all the ways questions can be modified to respond to this math goal or the research.)

Mathematical Goal: Students use reasoning strategies to compare fractions.

Q1) Put the following fractions in order from smallest to largest. Explain your thinking.

\[ \frac{1}{8}, \quad \frac{1}{125}, \quad \frac{1}{13}, \quad \frac{1}{57} \]

About Q1: Q1 involves ordering 4 unit fractions with a range of magnitudes. Because of the magnitudes of the unit fractions, Q1 has the potential to elicit:
- Unit fraction reasoning
- Inappropriate whole number reasoning.

Strategies such as using visual models or common denominators are not reasonable given the relative magnitude of the fractions.

Q2) Sheila believes that the inequality below is a true statement. Is she correct or incorrect? Explain your reasoning.

\[ \frac{1}{5} + \frac{1}{4} + \frac{1}{1} > \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \]

About Q2: Q2 involves comparing familiar unit fractions in the context of adding unit fractions. It has the potential to elicit:
- Understanding of addition of unit fractions
- Use of unit fraction reasoning (e.g., fifths are smaller than fourths so three-fifths is less than three-fourths. Sheila is incorrect.)
- Use of visual model
- Possibly, inappropriate whole number reasoning (e.g., 15 > 12 so Sheila is correct.)

Q3) Which fraction is closer to 1/2? Explain your thinking.

\[ \frac{2}{4} \quad \text{or} \quad \frac{5}{12} \]

About Q3: Q3 involves comparing fractions with different denominators and different numerators to a benchmark. Because 3/4 is 1/4 greater than 1/2 and 5/12 is 1/12 less than 1/2, the problem has the potential to elicit:
- Benchmark reasoning
- Use of extended unit fraction reasoning
- Use of visual model
- Use of common denominators
- Possibly, inappropriate whole number reasoning.

OGAP Fraction Framework

Depending upon the strength of fractional reasoning students may move up and down between fractional, transitional, early transition, and non-fractional reasoning and strategies as they interact with new topics or new concepts (Pett, Laird, & Marsden (in press 2015)).

Fraction Problem Structures

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Reasoning Strategies

- Number sense
- Unit fraction
- Extended unit fraction
- Using visual models
- Benchmark
- Equivalence

Class of Fractions

- Different numerators, same denominators
- Same numerators, different denominators
- Different numerators, different denominators

Operations

- Estimation
- Efficient algorithm
- Impact of operation
- Equivalence

Types of Problems

- Requires interaction with a visual model
- An exact numerical answer is required
- An exact numerical answer is NOT required
- Contextual
- Non-contextual
- Supporting a claim
- Mathematical explanation required
- Multi-step problems
- Link equation to a contextual problem
- Link equation to visual model
- Extended multiple choice
- Impact of operations

OGAP Fraction Framework is based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

There are three major elements to the OGAP Fraction Framework that should be considered when analyzing student work or making instructional decisions:

1. Problem structures
2. Evidence in student work along a progression
3. Evidence of underlying issues or errors

This page identifies problem structures for fraction problems that can be used in planning a lesson or selecting or designing a task. The centerfold is a learning progression designed to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions and provide feedback to students. The back page contains some examples of engineered questions based on mathematics education research.

Consistent with the CCSSM the OGAP Fraction Progression uses visual models, equipartitioning, unit fraction understanding, equivalence, and properties of operations as means to developing understanding and fluency of fractions. Ultimately, fluency will enable students to engage in middle school topics that assume proficiency with fractions (e.g., proportionality, solving equations with fractional coefficients).

As students interact with new concepts, new structures, and more complex problem solving situations they may move back and forth between fractional, transitional, early transition, and non-fractional reasoning and strategies. This is important evidence to use for instructional decision-making. For example, a student may consistently find a fractional part of a set or area by physically partitioning a given visual model. However, when asked to find 3/4 of 164, students may revert to a non-fractional strategy.

To learn more about the mathematics education research underpinning the OGAP Fraction Framework read A Focus on Fractions: Bringing Research to the Classroom (Pett, Laird, & Marsden). OGAP references are found at www.ogapmath.com.
Fraction Progression

Note: The examples provided do NOT represent the full set of possible solutions that represent each level.

Accurately locates fractions on a number line of any length, compares and orders fractions using a range of strategies, finds equivalent fractions, and operates efficiently when solving mathematical and contextual problems

Magnitude Reasoning

Early Fractional Strategies

Non-Fractional

Whole number reasoning, not fractional reasoning

Incorrect operation given context

Underlying Issues/Errors

Rule, without understanding

Misinterprets model

Inappropriate whole number reasoning

Errors in:

Size of whole

Calculation

Property

Equation

Other

Stephanie and Paige are discussing the answer to 3 2/7 x 5/9. Stephanie said that the answer is more than 3 2/7. Paige said the answer is less than 3 2/7. Who is correct?

Stephanie

Paige

Whole number reasoning, not fractional reasoning

Bob ran 4 times this week. Each run was 3 3/4 miles long. How far did Bob run this week?

Distributive property

Bob ran 4 times this week. Each run was 3 3/4 miles long. How far did Bob run this week?

Evidence of fractional thinking, but reasoning or strategy is not efficient

Uses a fractional or transitional strategy (like partitioning visual models) or an operation appropriate for the situation, but the solution includes an error (e.g., partitioning, size of whole, concept error in part of problem)

Uses fractional or transitional strategy

Evidence of fractional thinking, but reasoning or strategy is not efficient

Evidence of fractional thinking, but reasoning or strategy is not efficient

Fractional Strategies

Transitional Strategies

Fraction Progression

Middle school topics and concepts in which rational number understanding and procedures are applied:

Area, Volume, Surface Area

Percents

Expressions and equations

Proportions

Rates

Similarity

Transformations

Probability

Functions

Scaling

Measures of Central Tendency

Others

As students learn new concept or interact with new structures or problem situations they may move back and forth across these levels.

Note: The examples provided do NOT represent the full set of possible solutions that represent each level.

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