



## OGAP Sample Problems: Contexts & Topics, and Problem Types (12.11.2019)

Important: The samples below do not include the full range of problem types.

**Ratio - referent explicit:** There are red and blue marbles in a bag. The ratio of red to blue marbles is 1 : 2. Sue opened the bag and found 12 red marbles. How many blue marbles are in the bag?

**Ratio - referent not given, but implied:** There are red and blue marbles in a bag. The ratio of red to blue marbles is 1 : 2. Sue opened the bag and found 12 red marbles. How many marbles are in the bag altogether?

**Density:** Karl says that Town A has more raccoons per square mile. Josh says Town B has more raccoons per square mile. Who is right? Justify your answer.

Town A	Town B
60 square miles	40 square miles
480 raccoons	380 raccoons

**Constant Speed:** A train is moving at a constant rate of speed. It has traveled 230 miles in 5 hours. At this rate how many miles will the train travel in 9 hours?

**Buy/Consume:** A 20-ounce box of Toasty Oats costs \$2.98. How much does Toasty Oats cost per ounce?

**Concentrations:** Mary mixed 5 cans of orange juice and 6 cans of water. Thomas mixed 6 cans of orange juice and 7 cans of water. Which drink has a stronger orange flavor?

**Measurement Conversions:** How many feet of material is 4.5 yards?

**Similarity:** The dimensions of 4 rectangles are given below. Which 2 rectangles are similar? Show your work

a) 2 inches x 3 inches   b) 4 inches x 5 inches   c) 6 inches x 9 inches   d) 6 inches x 10 inches

**Scale:** Richard enlarged a photo using a scale factor of 1.8. If the dimensions of the original photo were 5 inches by 7 inches, what are the dimensions of the enlarged photo?

**Probability:** Sue has a bag containing 3 black marbles and 5 red marbles. Mary has a bag containing 5 black marbles and 10 red marbles. Whose bag would you choose to have the best chance of picking a red marble?

**Percent:** Range of percent situations (problems involving part, whole, and percent, simple interest, tax, tip, discount, markups, commissions, percent increase, and percent error).

**Slope:** Sam made a graph of the speed (miles/hour) of the train. The slope of graph is 23. How fast is the train traveling?

**Sampling:** There are 320 students at Westport Middle School.. A random sample of 80 middle school students at Westport indicated that 32 students' favorite ice cream was chocolate. Based on these survey results, about how many Westport Middle School students' favorite ice cream would be chocolate? What factors might influence the accuracy of the results of the random sample in predicting the number of students whose favorite ice cream is chocolate?

**Rate Comparison:** A 20-ounce box of Toasty Oats costs \$2.98. A 16-ounce of Toasty Oats costs \$2.48. Which box of Toasty Oats is a better buy?

**Missing Value:** Bob's shower uses 12 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes?

**Qualitative:** Nick ran exactly the same number of laps around the school track yesterday as he ran today. If it took him less time to run the laps today than yesterday, his running speed today would be:

a) faster   b) slower   c) exactly the same   d) Not enough information. Explain your choice.

**Non-proportional:** Kim and Bob were running equally fast around a track. Kim started first. When she had run 9 laps, Bob had run 3 laps. When Bob had run 15 laps, how many laps had Kim run?



# OGAP Proportionality Framework

12.11.2019

*Depending upon the strength of proportional reasoning students may move back and forth between using proportional, transitional, and non-proportional strategies as they interact with different problem structures and problem contexts.*

(Cramer, Post & Currier, 1993; Karplus, Pulos & Stage, 1983; VMP OGAP, 2006 & 2007)

## PROBLEM STRUCTURES

Contexts & Topics	Problem Types	Multiplicative Relationships	Ratio Referent
Ratios	Ratio	Both integral	Implied
Density	Unit Rate	Both non-integral	Explicit
Constant Speed ( $d = rt$ )	Ratio and Rate comparisons	Some non-integral	
Buy/Consume	Missing Value		
Concentrations	Scale Factor	<b>Numbers</b>	<b>Ratio Relationships</b>
Measurement Conversions	Qualitative	All whole numbers	Part: Whole
Similarity	Non-proportional	Rational numbers	Part: Part
Scale		Magnitude	
Probability			<b>Representations</b>
Percents	<b>Internal Structure</b>	<b>Language</b>	Coordinate Graph
Slope	Parallel	Natural (e.g., every)	Table
Sampling	Non-parallel	Mathematical (e.g., per)	Visual Model
			Expression
			Equation

Go to [www.ogapmathllc.com](http://www.ogapmathllc.com) for more information about OGAP

## ABOUT OGAP FRAMEWORKS

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

There are three major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions:

- 1) Problem structures
- 2) Evidence in student work along a progression
- 3) Evidence of underlying issues or errors

This page identifies contexts & topics, problem types, and problem structures for ratio and proportion problems. The centerfold is a learning progression to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions. Sample OGAP questions reflecting different contexts & topics, and types of problems are shown on Page 4.

For students to become strong in their application of ratios, rates, and proportional relationships they must interact with a range of problem contexts and problem structures. Beginning in grade 4 the CCSSM engages students in multiplicative change problems which set the foundation for proportional thinking. In grade 6 students focus on understanding ratios and unit rates and applying these concepts in problems involving unit pricing, constant rate of speed, measure conversions, and percent. In grade 7 this work is extended to proportional relationships and solving multi-step proportion problems, percent problems, scale problems, and developing probability. In grade 8 students study the connections between proportional relationships, graphs, and linear equations including understanding of similarity.

Consistent with the CCSSM the OGAP Frameworks on the centerfold shows a progression from using models, building up and down, and ratio tables to efficient and generalized strategies identified in Proportional Strategies in the OGAP Proportionality Framework.

As students interact with new concepts, new problem contexts, new structures, and more complex problem solving situations the strategies that they use may move back and forth between using proportional, transitional, and non-proportional strategies. This is important evidence to use for instructional decision making. For example, a student may consistently solve problems involving unit rate using a proportional strategy. However, when given a density comparison problem students may revert to a non-proportional strategy.



Uses  $y = kx$  (either symbolic or graphical representation moving between graphs, tables, and equations)

Bob's shower uses 14 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 8 minutes?

$$y = kx$$

$$y = 4\frac{2}{3} \text{ gal./min} \times 8 \text{ min}$$

$$y = 37\frac{1}{3} \text{ gallons}$$

Sets up a proportion and uses cross products

$$\frac{3 \text{ minutes}}{14 \text{ gallons}} = \frac{8 \text{ minutes}}{x}$$

$$14 \text{ gallons} \times 8 \text{ min} = 112 \quad 112 \div 3 \text{ minutes} = 37\frac{1}{3} \text{ gallons}$$

Uses the correct *ratio referent* in a ratio problem and applies multiplicative relationship

There are red and blue marbles in a bag. The ratio of red to blue marbles is 1:2. Sue opens the bag and found 12 red marbles. How many marbles are in the bag altogether?

$$\frac{\text{Red}}{\text{Blue}} = \frac{1}{2}$$

$$\frac{\text{Red}}{\text{whole Bag}} = \frac{1}{3} = \frac{12}{x}$$

$$x = 36 \text{ marbles in the bag}$$

Compares fractions, rates, and ratios

The dimensions of 4 rectangles are given below. Which 2 rectangles are similar?

- A) 2" X 8"
- B) 4" X 10"
- C) 6" X 12"
- D) 6" X 15"

$$\frac{\text{length } 2}{\text{width } 8} = \frac{1}{4}$$

$$\frac{\text{length } 4}{\text{width } 10} = \frac{2}{5}$$

$$\frac{\text{length } 6}{\text{width } 12} = \frac{1}{2}$$

$$\frac{\text{length } 6}{\text{width } 15} = \frac{2}{5}$$

Applies multiplicative relationship (scale factor)

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes?

$$\frac{18 \text{ gallons}}{3 \text{ min}} \times 4\frac{1}{3} = 78 \text{ gal}$$

If Bob takes a 13 min shower he will use 78 gal of water.

$$3 \overline{) 13} = 4\frac{1}{3}$$

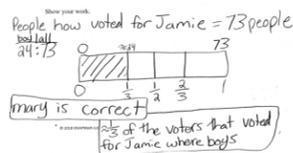
Finds and applies unit rate

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes?

$$\frac{6 \text{ gal}}{1 \text{ min}} \times 13 = 78 \text{ gal}$$

Uses a visual model multiplicatively

Dana and Jamie ran for student council president at Midvale Middle School. Twenty-four boys voted for Jamie. Forty-nine girls voted for Jamie. John said, "About half the students who voted for Jamie were boys." Mary said, "About one-third of the students voted for Jamie. Who is correct?"



Uses multiplicative relationship in ratio table

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes?

MIN	Gallons
3	18
6	36
9	54
12	72
13	78

Bob will use 78 gallons of water for a 13 minute shower.

Uses the multiplicative relationship for part of the problem

Paul's dog eats 20 pounds of food in 30 days. How long will it take Paul's dog to eat 45 pounds of dog food?

$$\frac{20 \text{ lb}}{30 \text{ days}} = \frac{45 \text{ lb}}{x \text{ days}}$$

$$45 = 30 \times 2.5$$

$$45 = 20 \times 2.5$$

Starts building up or repeated addition, but finds and applies multiplicative relationship

Bob's shower uses 18 gallons of water every three minutes. How many gallons of water does he use for a 13 minute shower?

$$\frac{18 \text{ gal}}{3 \text{ min}} + \frac{18 \text{ gal}}{3 \text{ min}} + \frac{18 \text{ gal}}{3 \text{ min}} + \frac{18 \text{ gal}}{3 \text{ min}} = \frac{72 \text{ gal}}{12 \text{ min}} + \frac{6 \text{ gal}}{1 \text{ min}} = \frac{78 \text{ gal}}{13 \text{ min}}$$

$18 \div 3 = 6 \text{ gal per min}$

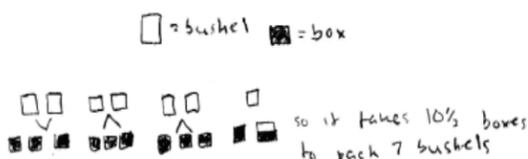
Builds up or down using multiplication

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes?

gallon	minute
18	3
36	6 x 2
72	12 x 2

Iterating composed units by repeatedly adding or by using skin counting

Carrie is packing apples for an orchard's mail order business. It takes 3 boxes to pack 2 bushels of apples. How many boxes will she need to pack 7 bushels of apples?



The ratio of red marbles to blue marbles in a bag is 1:2. Sue opened the bag and found 12 red marbles. How many marbles are in the bag?



Builds up or down using addition

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes?

g	m
18	3
+18	6 + 3
+18	9 + 3
+18	12 + 3
72	12

Uses additive difference, not multiplication relationship

Bob's shower uses 14 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 8 minutes?

$$\begin{array}{r} +5 \\ 3 \text{ min} \quad 8 \text{ min} \\ \hline 14 \text{ gal} \quad 19 \text{ gal} \end{array}$$

Uses incorrect *ratio referent* and makes an additive comparison

Compares numbers, not ratios or rates

Karl says that Town A has more raccoons per square mile. Josh says that Town B has more raccoons per square mile. Who is right? Justify your answer.

Town A	Town B
60 square miles	40 square miles
480 raccoons	380 raccoons

$$\frac{480}{60} = 8$$

$$\frac{380}{40} = 9.5$$

420 is bigger than 340 so Town A has more raccoons

Guesses or uses random operations      Misinterprets vocabulary and/or related concept  
Solves a non-proportional problem using a proportion

## Underlying Issues and Errors

Remainder error      Equation error      Missing or inconsistent use of units  
Misinterprets the meaning of quantities      Computation error      Rounding error