Using Learning Trajectories to Enhance Formative Assessment

Caroline B. Ebby and Marjorie Petit

Numerous research studies have shown that formative assessment is a classroom practice that when carried out effectively can improve student learning (Black and Wiliam 1998). Formative assessment is not just giving tests and quizzes more frequently. When assessment is truly formative, the evidence that is generated is interpreted by the teacher and the student and then used to make adjustments in the teaching and learning process. In other words, the formative assessment generates feedback, and that feedback is used to enhance student learning. Formative assessment is therefore fundamentally an interpretive process: It is less about the structure, format, or timing of the assessment and more about the function and use by both the teacher and student (Wiliam 2011). For teachers of mathematics, the heart of this process is making sense of and understanding student thinking in relation to content goals.

This is where research on student learning of mathematics can support the formative assessment process. Mathematics educators have made great progress over the last decades with respect to conducting and synthesizing this research in the form of learning trajectories. (*Learning trajectories* are also often referred to as *learning progressions*, particularly in science education.) A learning trajectory describes the progression of student thinking and strategies over time in terms of sophistication of both conceptual understanding and procedural fluency. For example, when students first learn a concept like multiplication, their strategies are often concrete and inefficient; they may draw equal groups and count objects one by one to find the total. Over time, as they learn to unitize single quantities into groups and develop place-value understanding, students might use repeated addition of the groups, skip counting, or open area models. Eventually these strategies are replaced with strategies that are more abstract and efficient and that draw on the properties of operations and algorithms. Understanding this developmental process in terms of both conceptual understanding and efficiency allows teachers both to assess where a student's understanding is on the trajectory and what the next instructional step should be to move the student's understanding forward. Learning trajectories describe the pathway to get from the student's prior knowledge to the mathematical goals or standards (Daro, Mosher, and Corcoran 2011).

Currently, learning trajectories exist in the research literature for many

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mathematical domains, including counting, addition and subtraction, multiplicative thinking, measurement, geometry, proportional reasoning, and algebraic thinking (Daro, Mosher, and Corcoran 2011). These trajectories are at the heart of the Common Core Standards and elucidated in the companion Progressions documents (Common Core Standards Writing Team 2013). In this article, we describe tools and routines developed by the Ongoing Assessment Project (OGAP) that make these trajectories accessible for teachers and can enhance both instructional practice and student learning. In particular, we illustrate how the OGAP Proportionality Progression can help teachers draw on this research to make sense of and respond to student thinking about proportional reasoning in the middle grades. (For more information on OGAP, see http://margepetit.com.)

USING LEARNING TRAJECTORIES TO INTERPRET AND RESPOND TO STUDENT WORK

For students to develop proportional reasoning in the middle grades, they must make a fundamental shift from additive to multiplicative reasoning across a range of proportional situations with varied problem structures. Research-based learning trajectories for proportional reasoning indicate that student strategies get more efficient over time as their understanding of multiplicative relationships develops and deepens.

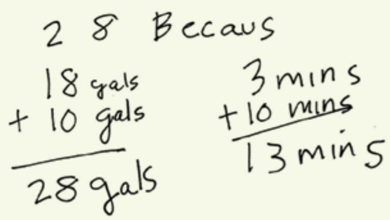
Figure 1 shows three different student responses to a missing-value proportion problem involving a rate. As you study the solution strategies, look for evidence of additive and multiplicative reasoning.

These three student responses illustrate different solution strategies that in turn reflect different levels of proportional reasoning along a progression. Response A illustrates incorrect additive reasoning: The student added 10 to each quantity to reach 13 minutes, without any evidence of recognizing the multiplicative nature of the rate in the problem situation. Response B shows evidence of both repeated addition and multiplicative thinking: The student repeatedly added the ratio of 18 gallons to 3 minutes to determine the gallons of water used for 12 minutes, then used division to find a unit rate and added that on to find how many gallons would be used for an additional minute. Although this building-up strategy is mathematically correct and shows understanding of proportions, it is a relatively inefficient method. Response C shows evidence of finding the multiplicative relationships both within (× 6) and between (× 4 1/3) the ratios and then correctly applying the multiplicative relationships to determine the number of gallons used after 13 minutes. (See **fig. 2** for a discussion of *within ratios* and *between ratios*.) The student also seemed to recognize midstream that using the within ratio would be more efficient, given the numbers in the problem (18 is evenly divisible by 3, and 13 divided by 3 yields a repeating decimal).

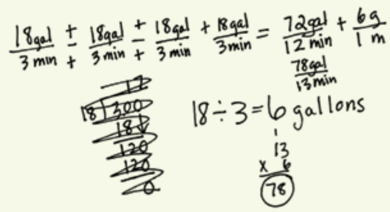
Figure 3 shows how the OGAP Proportionality Progression illustrates the development of student strategies from nonproportional to proportional

Fig. 1 These student solutions to a missing-value proportion problem reflect different levels of proportional reasoning.

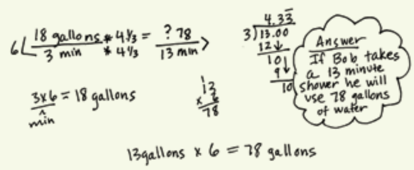
Bob's shower uses 18 gallons of water every 3 minutes. How many gallons of water does Bob use if he takes a 13-minute shower? Show your work.



Student Response A



Student Response B



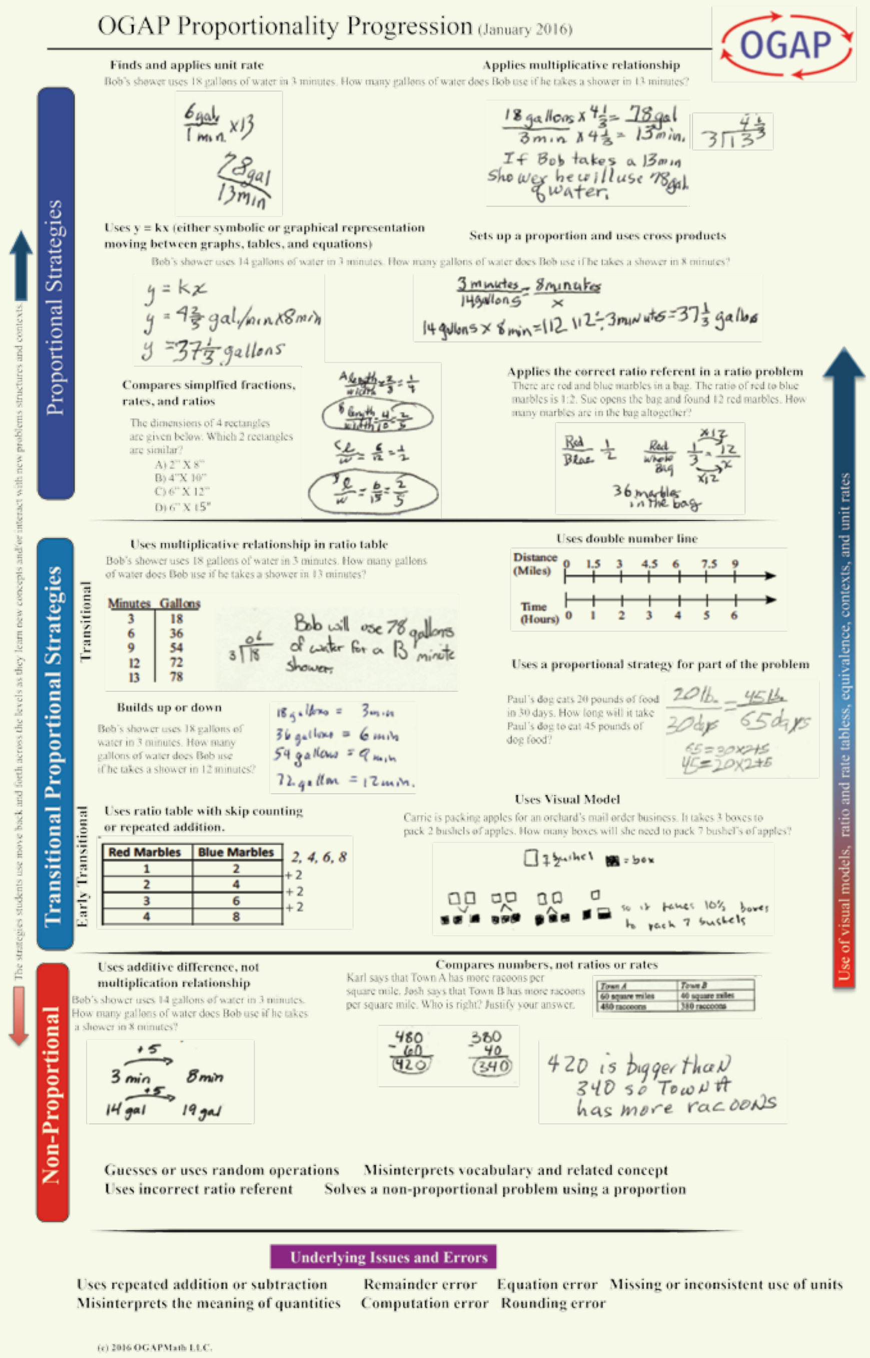
Student Response C

Fig. 2 Defining within and between ratios

$$\text{within ratios} \updownarrow \frac{3}{2} = \frac{x}{8}$$
$$\longleftrightarrow$$
between ratios

The terms "within ratios" and "between ratios" provide language to discuss the multiplicative relationships in a proportion. In the diagram above, the multiplicative relationships "within" are × 1.5 (bottom to top of ratio) and × 2/3 (top to bottom of ratio). The multiplicative relationships "between" the ratios are × 4 (left ratio to right ratio) and × 1/4 (right ratio to left ratio).

Fig. 3 The OGAP Proportionality Progression (Petit, Laird, and Hulbert 2016) can help teachers respond to student thinking.



Proportional Strategies are efficient and generalizable and include identifying or using the scale factor ($y = kx$) in tables, equations, and graphs, finding and applying a unit rate; using multiplicative relationships within or between rates; comparing simplified fractions, rates, or ratios; or setting up a proportion and using cross-products.

Transitional Strategies reflect multiplicative reasoning with the use of representations (e.g., a double number line, ratio tables), use proportional reasoning for only part of the problem, or are inefficient. Early transitional strategies rely on visual models, doubling or building up (repeated addition) strategies.

Nonproportional Strategies include the use of additive or inappropriate whole-number reasoning, guessing or using the incorrect operation, or misinterpreting vocabulary or related concepts.

Underlying Issues and Errors: Such issues in student work as computational errors, additive reasoning, unreasonable answers, equation errors, or misinterpretation of units should be noted, along with evidence about the level on the progression.

reasoning using examples of student strategies that a teacher might encounter from viewing student work. The transitional stage includes the use of visual models and specific representations (e.g., ratio tables, double number lines) that can help students build on understanding the under-lying multiplicative relationships in proportional situations to generate more efficient and generalizable proportional strategies. These transitional strategies are important in ensuring that students are not just developing rote or procedural understanding of algorithms. The arrow to the left of the levels on the OGAP Proportionality Progression reflects an important point about movement along a learning trajectory. That is, student strategies and reasoning move up and down between levels on the progression as students interact with new problem contents or new problem structures until proportional reasoning stabilizes (Cramer, Post, and Currier 1993).

This progression was designed at a granular level that makes it usable by teachers across a range of topics that involve proportional reasoning (e.g., ratios, similarity, percentage). Its purpose is to provide teachers with descriptive evidence of student understanding and student progress toward using efficient and generalizable strategies. This analysis is meant to inform instruction and learning from a formative, not evaluative, perspective. Teachers use the progression to sort the evidence in student work by the levels along the progression, rather than starting with correctness or accuracy. This sorting allows teachers to focus on the strategies that students use and the evidence of underlying understanding and proportional reasoning. Next, teachers review each stack to see if there are any errors (e.g., calculation errors) or underlying issues (e.g., missing or incorrect units). This kind of evidence provides information to inform instruction.

For example, because student A's solution reflects additive reasoning, the student could benefit from learning to represent equivalent ratios in a model, such as the ratio table, tape diagram, or double number line. A teacher might use student B's solution to highlight the multiplicative relationships in the equivalent ratios that are generated. Likewise, student C's solution can be used to talk about why it is easier to use the within relationship in this problem. By analyzing these strategies in relation to the learning progression, teachers can adapt instruction to build on student thinking and help develop procedural fluency with conceptual understanding.

The OGAP Progression is part of a larger OGAP Proportionality Framework that also describes the range of problem contexts (e.g., rates, scaling, similarity) and structures (e.g., complexity of numbers, internal structure of problems, the multiplicative relationships) that students encounter as they develop their proportional reasoning (Petit, Laird, and Hulbert 2016). Ultimately, a student who has strong proportional reasoning should not be influenced by context, problem types, the quantities in the problems and their associated units, or numerical complexity (Cramer, Post, and Currier 1993).

Strong evidence exists that knowledge and instructional use of research-based frameworks of how students build mathematical understanding enhances instructional decision making, leading to gains in both student motivation and achievement (e.g., Carpenter et al. 1989; Clements et al. 2011). The coupling of formative assessment and learning trajectories provides teachers and students with powerful tools to inform instruction and learning, thereby shifting from assessment of learning to assessment for learning (Stiggins and Chappuis 2006).

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