About the OGAP Item Bank: The OGAP multiplicative reasoning item bank is composed of 280 short constructed response or extended multiple choice items that assess multiplication and division in the CCSS for grades 3-5. Each question is strategically engineered considering the interaction of the context of the problem and structures of problems (e.g., property, complexity of factors or divisors) that will elicit student developing understanding, common errors that students make, or preconceptions or misconceptions that may interfere with students learning new concepts or solving related problems. Some problems focus specifically on multiplicative or division while others involve multistep operations. The multi-step problems are coded in this document as "multiplication and division" and "multiplication or division and addition or subtraction." Pages 18-19 lists items by problem contexts and CCSSM clusters. (The Volume items are PILOT and are shaded.)

There are two types of problem in the OGAP item bank:

1) Application/context: Application/context problems are designed to elicit the strategies that students use to solve multiplication and division problems. (e.g., counting by ones, area model, algorithm)
2) Concept/property: Concept/property problem target very specific concept understanding (e.g., the base ten system, equations) or properties of operations (e.g., commutative, distributive). These items are found primarily in the Property and Equation sets of items.

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## Property (page 1 of 2 )

| $\begin{array}{\|c\|} \hline \text { Item Bank } \\ \text { ID } \end{array}$ | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | $\begin{gathered} \text { Complexity of } \\ \text { Divisors } \end{gathered}$ | Number of factors | Properties | Property | Property | Representatio ns | Relationships |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PR1 | Concept/prope rty | Equations | Multiplication |  | Single digit x multidigit |  |  | Distributive |  |  | Arrays | Modelequation |
| PR3 | Concept/prope rty | Equal groups | Multiplication |  | Single digit x single digit |  |  | Distributive |  |  |  | Contextequation |
| PR4 | Concept/prope rty | Properties | Multiplication and Division |  | Single digit x single digit | Single digit |  | Multiplicative inverse |  |  |  |  |
| PR5 | Concept/prope rty | Equations | Multiplication |  | Single digit x multidigit |  |  | Distributive |  |  | Arrays | Modelequation |
| PR7 | Concept/prope rty |  | Multiplication |  | Single digit x multidigit |  | 2 |  |  |  |  | Doubling and halving |
| PR9 | Concept/prope rty |  | Multiplication |  | Single digit x single digit |  |  | Distributive |  |  |  | Doubling and halving |
| PR10 | Concept/prope rty | Equal groups | Multiplication |  | Single digit x single digit |  |  | Commutative |  |  |  |  |
| PR11 | Concept/prope rty |  | Multiplication |  | Single digit x single digit |  |  | Commutative |  |  |  |  |
| PR12 | Concept/prope rty |  | Multiplication |  | Single digit x single digit |  | More than 2 | Associative |  |  |  |  |
| PR13 | Concept/prope rty |  | Multiplication |  | Single digit x single digit |  | More than 2 | Associative |  |  |  |  |
| PR15 | Concept/prope rty | Equations | Division |  |  | Single digit |  | Commutative |  |  |  |  |
| PR18 | Concept/prope rty |  | Multiplication |  | Powers of ten |  |  |  |  |  |  | Multiples and factors |
| PR19 | Concept/prope rty |  | Division |  |  | Powers of ten |  |  |  |  |  | Multiples and factors |
| PR21 | Concept/prope rty |  | Multiplication |  | Multi-digit x multidigit |  |  | Distributive |  |  |  |  |
| PR23 | Concept/prope rty |  | Division |  |  | Single digit |  | Distributive |  |  |  |  |
| PR24 | Concept/prope rty |  | Division |  |  | Single digit |  |  |  |  |  | Doubling and halving |
| PR25 | Concept/prope rty |  | Multiplication |  | Multi-digit x multidigit |  |  | Distributive |  |  | Open area | Modelequation |
| PR30 | Concept/prope rty | Properties | Multiplication |  | Powers of ten |  |  |  |  |  |  |  |
| PR31 | Concept/prope rty | Properties | Multiplication |  | Powers of ten |  |  |  |  |  |  |  |

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## Property (page 2 of 2)



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Volume (Pilot items)

| $\begin{array}{\|c\|} \hline \text { Item Bank } \\ \text { ID } \end{array}$ | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V1 | Concept/prope rty | Volume | Multiplication |  | Single digit $x$ multi-digit |  | More than 2 |
| V2 | Concept/prope rty | Volume | Multiplication |  |  |  | More than 2 |
| V3 | Concept/prope rty | Volume | Multiplication and Division |  | Single digit $x$ multi-digit |  | More than 2 |
| V4 | Application/co ntext | Volume | Multiplication |  | Single digit $x$ multi-digit |  | More than 2 |
| V5 | Concept/prope rty | Volume | ```Addition/subtraction with multiplication/divisio n``` |  | Single digit $x$ multi-digit |  | More than 2 |
| V6 | Application/co ntext | Volume | Multiplication |  | Multi-digit x multi-digit |  | More than 2 |
| V7 | Application/co ntext | Volume | with multiplication/divisio |  | Multi-digit x multi-digit |  | More than 2 |
| V8 | Application/co ntext | Volume | Multiplication and Division |  | Multi-digit x multi-digit |  | More than 2 |
| V9 | Application/co ntext | Volume | ```Addition/subtraction with multiplication/divisio n``` |  | Multi-digit x multi-digit |  | More than 2 |
| V10 | Application/co ntext | Volume | Multiplication |  | Fraction/decim als |  | More than 2 |
| V11 | Application/co ntext | Volume | Multiplication and Division |  | Multi-digit x multi-digit |  | More than 2 |
| V12 | Application/co ntext | Volume | Multiplication and Division |  | Fraction/decim als |  | More than 2 |
| V13 | Application/co ntext | Volume | Multiplication and Division |  | Fraction/decim als |  | More than 2 |

[^0]
## Equal Measure

| $\begin{array}{\|c\|} \hline \text { Item Bank } \\ \text { ID } \\ \hline \end{array}$ | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors | Properties | Representatio ns | Relationships | Problem situations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| EM1 | Application/co ntext | Equal measures | Division | Quotative |  | Multi-dgit |  |  |  |  | Number of groups unknown |
| EM2 | Application/co ntext | Equal measures | Division | Partitive |  | Single digit |  |  |  |  | Group size unknown |
| EM3 | Concept/prope rty | Equal measures | Division | Quotative |  |  |  |  |  |  |  |
| EM4 | Application/co ntext | Equal measures | ```Addition/subtraction with multiplication/divisio n``` |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| EM6 | Application/co ntext | Equal measures | Multiplication and Division |  | Single digit x single digit |  | 2 |  |  |  | Number of groups unknown |
| EM8 | Application/co ntext | Equal measures | Multiplication |  | Single digit x multi-digit |  | 2 |  |  |  | Unknown total |
| EM11 | Application/co ntext | Equal measures | Division | Quotative |  | Single digit |  |  |  | Meaning of remainders | Number of groups unknown |
| EM12 | Application/co ntext | Equal measures | Multiplication |  | Single digit $x$ multi-digit |  |  |  |  |  | Unknown total |
| EM13 | Application/co ntext | Equal measures | Multiplication |  | Multi-digit x multi-digit |  | 3 |  |  |  | Unknown total |
| EM14 | Application/co ntext | Equal measures | Division | Quotative |  | Multi-dgit |  |  |  | Meaning of remainders | Number of groups unknown |
| EM15 | Application/co ntext | Equal measures | Addition/subtraction with multiplication/divisio n |  | Fraction/decim als |  |  |  |  |  | Unknown total |
| EM16 | Application/co ntext | Equal measures | Division | Quotative |  | Fractions/deci mals |  |  |  | Meaning of remainders | Number of groups unknown |
| EM17 | Application/co ntext | Equal measures | ```Addition/subtraction with multiplication/divisio n``` | Quotative |  | Powers of ten |  |  |  |  |  |
| EM18 | Application/co ntext $\qquad$ | Equal measures | Multiplication |  | Multi-digit x multi-digit |  |  |  |  |  | Unknown total |

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## Equal Groups (page 1 of 4)

| Item Bank ID | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors | Properties | Representatio ns | Relationships | Problem situations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| EG1 | Application/co ntext | Equal groups | Multiplication and Division | Quotative | Single digit x multi-digit |  | 2 |  |  |  | More than one situation |
| EG2 | Application/co ntext | Equal groups | Division | Quotative |  | Single digit |  |  |  | Meaning of remainders | Number of groups unknown |
| EG3 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG4 | Application/co ntext | Equal groups | Division | Quotative |  | Single digit |  |  |  |  | Number of groups unknown |
| EG5 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | More than 2 |  |  |  | Unknown total |
| EG6 | Application/co ntext | Equal groups | Multiplication |  | Single digit x single digit | G | More than 2 |  |  |  | Unknown total |
| EG7 | Concept/prope rty | Equal groups | Multiplication |  |  |  |  |  | Equal groups |  |  |
| EG8 | Concept/prope rty | Equal groups | Multiplication |  | Single digit $x$ single digit |  | 2 |  | Equal groups |  |  |
| EG9 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG10 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG11 | Application/co ntext | Equal groups | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| EG12 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG13 | Application/co ntext | Equal groups | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| EG14 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG15 | Application/co ntext | Equal groups | Addition/subtraction with multiplication/divisio n |  | Single digit x single digit |  | 2 |  |  |  | Unknown total |
| EG16 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG17 | Application/co ntext | Equal groups | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  | Doubling and halving | Unknown total |
| EG18 | Application/co ntext | Equal groups | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| EG19 | Application/co ntext | Equal groups | Addition/subtraction with multiplication/divisio n |  | Single digit $x$ multi-digit |  |  | Distributive |  |  | Unknown total |

Equal Group (page 2 of 4)

| EG20 | Application/co ntext | Equal groups | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| EG21 | Application/co ntext | Equal groups | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| EG22 | Application/co ntext | Equal groups | Multiplication |  | Single digit x multi-digit |  | 2 |  |  |  | Unknown total |
| EG24 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG25 | Application/co ntext | Equal groups | Multiplication |  | Powers of ten |  | 2 |  |  |  | Unknown total |
| EG26 | Application/co ntext | Equal groups | Multiplication |  | Single digit x single digit |  | 2 |  |  |  | Unknown total |
| EG27 | Application/co ntext | Equal groups | Multiplication |  | Powers of ten |  | 2 |  |  |  | Unknown total |
| EG28 | Application/co ntext | Equal groups | Multiplication |  | Single digit x multi-digit |  | More than 2 |  |  |  | Unknown total |
| EG29 | Application/co ntext | Equal groups | Multiplication |  | Powers of ten |  | More than 2 |  |  |  | Unknown total |
| EG30 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG31 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| EG32 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  |  | Unknown total |
| EG33 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ single digit |  | More than 2 |  |  |  | Unknown total |
| EG34 | Application/co ntext | Equal groups | Multiplication |  | Single digit x multi-digit |  | More than 2 |  |  |  | Unknown total |
| EG35 | Application/co ntext | Equal groups | Multiplication |  | Single digit x single digit |  | 2 |  |  |  | Unknown total |
| EG36 | Concept/prope rty | Equal groups | Multiplication |  | Single digit x single digit |  | 2 |  | Equal groups |  | Unknown total |
| EG37 | Concept/prope rty | Equal groups | Addition/subtraction with multiplication/divisio n |  | Single digit x single digit |  | 2 |  |  |  | Unknown total |
| EG38 | Application/co ntext | Equal groups | Division | Quotative |  | Multi-dgit |  |  |  |  | Number of groups unknown |
| EG39 | Application/co ntext | Equal groups | Division | Quotative |  | multi-digit |  |  |  | Meaning of remainders | Number of groups unknown |

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## Equal Group (page 3 of 4)

| EG41 | Application/co ntext | Equal groups | Division | Quotative |  | Single digit |  |  |  | Number of groups unknown |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| EG42 | Application/co ntext | Equal groups | Division | Quotative |  | Multi-dgit |  |  |  | Number of groups unknown |
| EG43 | Application/co ntext | Equal groups | Division | Partitive |  | Single digit |  |  |  | Group size unknown |
| EG44 | Application/co ntext | Equal groups | Division | Partitive |  | Single digit |  |  |  | Group size unknown |
| EG45 | Application/co ntext | Equal groups | Division | Partitive |  | Single digit |  |  |  | Unknown total |
| EG46 | Application/co ntext | Equal groups | Division | Partitive |  | Single digit |  |  |  | Unknown total |
| EG47 | Application/co ntext | Equal groups | Division | Partitive |  | Single digit |  |  |  | Group size unknown |
| EG49 | Application/co ntext | Equal groups | Addition/subtraction with multiplication/divisio n |  | Single digit x single digit |  | 2 | Equal groups |  | Unknown total |
| EG50 | Application/co ntext | Equal groups | Addition/subtraction with multiplication/divisio n | Quotative |  | Single digit |  |  |  | Number of groups unknown |
| EG51 | Application/co ntext | Equal groups | Division | Quotative |  | Single digit |  |  |  | Number of groups unknown |
| EG52 | Application/co ntext | Equal groups | Division | Quotative |  | Single digit |  |  |  | Number of groups unknown |
| EG53 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  | Unknown total |
| EG54 | Application/co ntext | Equal groups | Multiplication |  | Single digit x single digit |  | 3 |  |  | Unknown total |
| EG55 | Application/co ntext | Equal groups | Addition/subtraction <br> with <br> multiplication/divisio <br> n | Quotative |  | Single digit |  |  | Meaning of remainders | Number of <br> groups <br> unknown |
| EG56 | Application/co ntext | Equal groups | Division | Quotative |  | Multi-dgit |  |  |  | Number of groups unknown |
| EG57 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  | Unknown total |
| EG58 | Concept/prope <br> rty | Equal groups | Multiplication |  | Single digit x multi-digit |  | 2 |  | Multiples and factors | Unknown product |
| EG59 | Application/co ntext | Equal groups | Multiplication |  | Single digit $x$ <br> single digit |  | More than 2 |  |  | Unknown total |
| EG60 | Application/co ntext | Equal groups | Division | Quotative |  | Powers of ten |  |  |  | Number of groups unknown |
| EG 62 | Application/co ntext | Equal groups | Multiplication |  | Single digit x single digit |  | 2 |  |  | Unknown total |

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## Equal Groups (page 4 of 4)

| EG 65 | Application/co ntext | Equal groups | Division | Partitive |  | Multi-dgit |  |  |  |  | Group size unknown |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| EG 66 | Application/co ntext | Equal groups | Division | Partitive |  | Multi-dgit |  |  |  | Meaning of remainders | Group size unknown |
| EG 67 | Application/co ntext | Equal groups | Multiplication and Division |  | Single digit x multi-digit |  |  |  |  |  | Unknown Total |
| EG 68 | Application/co ntext | Equal groups | Multiplication and Division |  | Single digit x multi-digit | Single digit |  |  |  |  | Unknown total |
| EG 69 | Application/co ntext | Equal groups | Multiplication |  | Single digit x single digit |  |  |  |  |  | Unknown total |
| EG70 | Application/co ntext | Equal groups | Division | Partitive |  | Multi-dgit |  |  |  | Meaning of remainders | Group size unknown |
| EG71 | Application/co ntext | Equal groups | Division | Quotative |  | Multi-dgit |  |  |  | Meaning of remainders | Number of groups unknown |
| EG72 | Application/co ntext | Equal groups | Division | Quotative |  | Multi-dgit |  |  |  | Meaning of remainders | Number of groups unknown |
| EG73 | Application/co ntext | Equal groups | Division | Partitive |  | Multi-dgit |  |  |  | Meaning of remainders | Group size unknown |
| EG74 | Application/co ntext | Equal groups | Division | Partitive |  | Multi-dgit |  |  |  | Meaning of remainders | Group size unknown |
| EG75 | Application/co ntext | Equal groups | Division | Partitive |  | Fractions/deci mals |  |  |  | Meaning of remainders | Group size unknown |

## Multiplicative Comparison

| $\begin{gathered} \text { Item Bank } \\ \text { ID } \\ \hline \end{gathered}$ | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors | Properties | Representatio ns | Relationships | Problem situations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MCM1 | Application/co ntext | Multiplicative comparison | Multiplication |  | Powers of ten |  | More than 2 |  |  |  | Unknown total |
| MCM2 | Application/co ntext | Multiplicative comparison | Multiplication and Division |  | Single digit $x$ single digit | Single digit | 2 |  |  |  | Group size unknown |
| MCM3 | Application/co ntext | Multiplicative comparison | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| MCM4 | Application/co ntext | Multiplicative comparison | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  |  | Unknown total |
| MCM5 | Application/co ntext | Multiplicative comparison | Division |  | Powers of ten | Multi-dgit | 2 |  |  |  | Number of groups unknown |
| MCM6 | Application/co ntext | Multiplicative comparison | Division |  |  | Single digit | 2 |  |  |  | Number of groups unknown |
| MCM7 | Application/co ntext | Multiplicative comparison | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| MCM8 | Application/co ntext | Multiplicative comparison | Multiplication |  | Base Ten |  | 2 |  |  |  | Unknown total |
| MCM11 | Application/co ntext | Multiplicative comparison | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  |  | Unknown total |
| MCM12 | Application/co ntext | Multiplicative comparison |  |  |  |  |  |  |  |  |  |
| MCM13 | Application/co ntext | Multiplicative comparison | Division |  |  | Single digit |  |  |  |  | Group size unknown |
| MCM15 | Application/co ntext | Multiplicative comparison | Division |  |  | Single digit |  |  |  |  | Number of groups unknown |
| MC16 | Application/co ntext | Multiplicative change | Division |  |  | Single digit |  |  |  |  | Group size unknown |
| MC17 | Application/co ntext | Multiplicative change | Division |  |  | Multi-dgit |  |  |  |  | Number of groups unknown |
| MC18 | Application/co ntext | Multiplicative change | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| MC19 | Application/co ntext | Multiplicative change | Multiplication |  | Single digit x <br> single digit |  | 2 |  |  |  | Unknown total |
| MC20 | Application/co ntext | Multiplicative change | Division |  |  | Powers of ten |  |  |  |  | Number of groups unknown |
| MC21 | Application/co ntext | Multiplicative change | Division |  |  | Powers of ten |  |  |  |  | Number of groups unknown |
| MC22 | Application/co ntext | Multiplicative change | Division |  |  | Powers of ten |  |  |  |  | Group size unknown |

## Patterns

| Item Bank ID | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors | Properties | Representatio ns | Relationships | Problem situations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P1 | Application/co ntext | Patterns | Multiplication |  | Single digit $x$ multi-digit |  |  |  |  |  |  |
| P2 | Application/co ntext | Patterns | Addition/subtraction with multiplication/divisio n |  | Single digit x <br> single digit |  |  |  |  |  |  |
| P3 | Application/co ntext | Patterns | Multiplication and Division | Partitive | Multi-digit X multi-digit | Single digit | 2 |  |  |  | More than one situation |
| P4 | Application/co ntext | Patterns | Multiplication and Division | Partitive | Single digit $x$ multi-digit | Single digit | 2 |  |  |  | More than one situation |
| P5 | Application/co ntext | Patterns | Multiplication and Division |  | Single digit $x$ single digit |  |  |  |  | Contextequation |  |
| P6 | Application/co ntext | Patterns | Multiplication and Division |  | Single digit $x$ multi-digit |  |  |  |  | Contextequation |  |
| P7 | Application/co ntext | Patterns | Division | Quotative |  | Multi-dgit |  |  |  |  | Number of groups unknown |
| P8 | Application/co ntext | Patterns | Division |  |  | Multi-dgit |  |  |  | Contextequation |  |
| P9 | Application/co ntext | Patterns | Division |  |  | Single digit |  |  |  | Contextequation |  |
| P10 | Application/co ntext | Patterns | Division |  |  | Multi-dgit |  |  |  | Contextequation |  |
| P11 | Application/co ntext | Patterns | Multiplication and Division |  | Single digit $x$ single digit | Single digit |  |  |  | Contextequation |  |
| P12 | Application/co ntext | Patterns | Multiplication and Division |  | Single digit $x$ single digit | Single digit |  |  |  | Contextequation |  |
| P13 | Application/co ntext | Patterns | Addition/subtraction with multiplication/divisio n |  |  |  |  |  |  |  |  |
| P14 | Application/co ntext | Patterns | Addition/subtraction with multiplication/divisio n |  |  |  |  |  |  |  |  |
| P15 | Application/co ntext | Patterns | Multiplication |  | Powers of ten |  |  |  |  |  |  |
| P16 | Application/co ntext | Patterns | Multiplication |  | Fraction/decim als |  |  |  |  | Context -model |  |
| P17 | Application/co ntext | Patterns | Multiplication |  | Single digit $x$ single digit |  |  |  |  |  |  |

## Rate

| $\begin{array}{\|c\|} \hline \text { Item Bank } \\ \text { ID } \end{array}$ | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors | Properties | Representatio ns | Relationships | Problem situations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R1 | Application/co ntext | Rates | Multiplication |  |  |  |  |  |  |  |  |
| R2 | Application/co ntext | Rates | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  | Unknown total |
| R3 | Application/co ntext | Rates | Multiplication |  | Fraction/decim als |  | 2 |  |  |  | Unknown total |
| R4 | Application/co ntext | Rates | Division |  |  | Single digit |  |  |  |  | Group size unknown |
| R5 | Application/co ntext | Rates | Division |  |  | Multi-dgit |  |  |  |  | Group size unknown |
| R6 | Application/co ntext | Rates | Division |  |  | Single digit |  |  |  |  | Group size unknown |
| R7 | Application/co ntext | Rates | Division |  |  | Multi-dgit |  |  |  |  | Number of groups unknown |
| R8 | Application/co ntext | Rates | Division |  |  | Single digit |  |  |  |  | Number of groups unknown |
| R9 | Application/co ntext | Rates | Division |  |  | Single digit |  |  |  |  | Number of groups unknown |
| R10 | Application/co ntext | Rates | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| R11 | Application/co ntext | Rates | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  |  | Unknown total |
| R12 | Application/co ntext | Rates | Division |  |  | Multi-dgit |  |  |  |  | Number of groups unknown |
| R13 | Application/co ntext | Rates | Multiplication |  | Multi-digit x multi-digit |  | 2 |  |  |  | Unknown total |
| R14 | Application/co ntext | Rates | Division |  |  | Multi-dgit |  |  |  |  | Number of groups unknown |
| R15 | Application/co ntext | Rates | Division |  |  | Multi-dgit |  |  |  |  | Number of groups unknown |
| R16 | Application/co ntext | Rates | Multiplication |  | Single digit x multi-digit |  | 2 |  |  |  | Unknown total |
| R17 | Application/co ntext | Rates | Division |  |  | Single digit | 2 |  |  |  | Group size unknown |
| R18 | Application/co ntext | Rates | Multiplication |  | Single digit x multi-digit |  | 2 |  |  |  | Unknown total |
| R19 | Application/co ntext | Rates | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  |  | Unknown total |
| R20 | Application/co ntext | Rates | Division |  |  | Single digit |  |  |  |  | Group size unknown |

## Rectangular Area

| $\begin{array}{\|c\|} \hline \text { Item Bank } \\ \text { ID } \\ \hline \end{array}$ | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors | Properties | Representatio ns | Relationships | Problem situations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RA1 | Application/co ntext | Area | Multiplication |  | Single digit x multi-digit |  | 2 |  | Open area |  |  |
| RA2 | Application/co ntext | Area | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  | Contextequation |  |
| RA3 | Application/co ntext | Area | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  | Open area | Contextequation |  |
| RA4 | Application/co ntext | Area | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  |  |  |
| RA5 | Application/co ntext | Area | Multiplication |  | Single digit $x$ multi-digit |  | More than 2 |  | Area |  |  |
| RA6 | Application/co ntext | Area | Multiplication |  | Single digit x single digit |  | 2 |  | Area |  |  |
| RA10 | Application/co ntext | Area | Multiplication |  | Single digit x single digit |  | 2 |  | Area |  |  |
| RA12 | Application/co ntext | Area | Division |  |  | Single digit |  |  |  |  |  |
| RA13 | Application/co ntext | Area | ```Addition/subtraction with multiplication/divisio n``` |  | Single digit x multi-digit | Multi-dgit | 2 |  | Open area |  |  |
| RA14 | Application/co ntext | Area | Division |  |  | Multi-dgit |  |  |  | Contextequation |  |
| RA15 | Application/co ntext | Area | Multiplication |  | $\begin{array}{\|l} \hline \text { Multi-digit x } \\ \text { multi-digit } \\ \hline \end{array}$ |  | 2 |  |  |  |  |
| RA16 | Application/co ntext | Area | Multiplication |  | Single digit $x$ single digit |  | 2 |  | Area |  |  |
| RA17 | Application/co ntext | Area | Addition/subtraction with multiplication/divisio n |  | Multi-digit x multi-digit |  | 2 |  | Open area |  |  |
| RA19 | Application/co ntext | Area | ```Addition/subtraction with multiplication/divisio n``` |  | Multi-digit x multi-digit |  | 2 |  | Open area |  |  |
| RA20 | Application/co ntext | Area | Division |  |  | Powers of ten |  |  | Open area | Contextequation |  |
| RA21 | Application/co ntext | Area | Multiplication and Division |  | $\begin{array}{\|l} \hline \text { Multi-digit x } \\ \text { multi-digit } \\ \hline \end{array}$ |  | 2 |  |  |  |  |
| RA22 | Application/co ntext | Area | ```Addition/subtraction with multiplication/divisio n``` |  | Fraction/decim als |  | 2 |  | Open area |  |  |
| RA23 | $\begin{array}{\|l\|} \hline \begin{array}{l} \text { Application/co } \\ \text { ntext } \end{array} \\ \hline \end{array}$ | Area | Multiplication |  | $\begin{array}{\|l\|} \hline \begin{array}{l} \text { Fraction/decim } \\ \text { als } \end{array} \\ \hline \end{array}$ |  | 2 |  |  |  |  |

## Measure Conversions (page 1 of 2)

| Item Bank ID | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MCV1 | Application/co ntext | Measure conversions | Multiplication |  | Multi-digit x multi-digit |  | 2 |
| MCV2 | Application/co ntext | Measure conversions | Multiplication |  | Single digit x single digit |  | 2 |
| MCV3 | Application/co ntext | Measure conversions | Multiplication |  | Multi-digit x multi-digit |  | 2 |
| MCV4 | Application/co ntext | Measure conversions | Multiplication |  | Single digit x single digit |  | 2 |
| MCV5 | Application/co ntext | Measure conversions | Division |  |  | Powers of ten | 2 |
| MCV6 | Application/co ntext | Measure conversions | Multiplication |  | Powers of ten |  | 2 |
| MCV7 | Application/co ntext | Measure conversions | Multiplication |  | Powers of ten |  | 2 |
| MCV8 | Application/co ntext | Measure conversions | Multiplication |  | Powers of ten |  |  |
| MCV9 | Application/co ntext | Measure conversions | Division |  |  | Single digit |  |
| MCV10 | Application/co ntext | Measure conversions | Division |  |  | Single digit |  |
| MCV11 | Application/co ntext | Measure conversions | Multiplication and Division |  | Single digit x multi-digit | Fractions/deci mals | 2 |
| MCV12 | Application/co ntext | Measure conversions | Multiplication |  | Powers of ten |  | 2 |
| MCV13 | Application/co ntext | Measure conversions | Division |  |  | Multi-dgit |  |
| MCV14 | Application/co ntext | Measure conversions | Multiplication and Division |  | Single digit x multi-digit | Multi-dgit | 2 |
| MCV15 | Application/co ntext | Measure conversions | Multiplication and Division |  | Single digit x <br> single digit | Single digit | 2 |
| MCV16 | Application/co ntext | Measure conversions | Multiplication |  | Single digit x single digit |  | 2 |
| MCV17 | Application/co ntext | Measure conversions | Division |  |  | Single digit |  |
| MCV18 | Application/co ntext | Measure conversions | Multiplication |  | Single digit x multi-digit |  | More than 2 |

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## Measure Conversions (page 2 of 2)

| MCV19 | Application/co ntext | Measure conversions | Multiplication | Single digit x multi-digit |  | 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MCV20 | Application/co ntext | Measure conversions | Division |  | Single digit |  |  |
| MCV21 | Application/co ntext | Measure conversions | Addition/subtraction with multiplication/divisio n | Single digit x multi-digit | Single digit | 2 |  |
| MCV22 | Application/co ntext | Measure conversions | Multiplication | Fraction/decim als |  | 2 |  |
| MCV23 | Application/co ntext | Measure conversions | Addition/subtraction with multiplication/divisio n | Powers of ten |  | 2 |  |
| MCV24 | Application/co ntext | Measure conversions | ```Addition/subtraction with multiplication/divisio n``` | Single digit x single digit |  | 2 |  |
| MCV25 | Application/co ntext | Measure conversions | Multiplication and Division | Single digit $x$ multi-digit | Powers of ten | 2 |  |
| MCV26 | Application/co ntext | Measure conversions | Multiplication | Single digit $x$ multi-digit |  | 2 |  |
| MCV27 | Application/co ntext | Measure conversions | Addition/subtraction with multiplication/divisio n | Powers of ten |  | 2 |  |
| MCV28 | Application/co ntext | Measure conversions | Addition/subtraction with multiplication/divisio n |  | Multi-dgit |  |  |
| MCV29 | Application/co ntext | Measure conversions | Multiplication | Single digit x multi-digit |  | More than 2 |  |
| MCV30 | Application/co ntext | Measure conversions | Multiplication | Powers of ten |  | More than 2 |  |
| MCV31 | Application/co ntext | Measure conversions | Multiplication and Division | Powers of ten |  | More than 2 |  |
| MCV32 | Application/co ntext | Measure conversions | Multiplication and Division | Multi-digit x multi-digit | Single digit |  |  |

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## Equations (page 1 of 2)

| $\begin{array}{\|c} \hline \text { Item Bank } \\ \text { ID } \\ \hline \end{array}$ | Problem Type | Problem Contexts | Operation | Types of Division | Complexity of Factors | Complexity of Divisors | Number of factors | Properties | Representatio $n s$ | Relationships | Problem situations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |
| E1 | Concept/property | Equations | Multiplication |  | Powers of ten |  |  | Distributive |  | Model-equation |  |
| E2 | Concept/property | Equations | Multiplication |  | Single digit x single digit |  | 2 |  | Arrays | Model-equation | Unknown total |
| E3 | Concept/property | Equations | Multiplication |  | Single digit x single digit |  | 2 |  |  |  | Unknown total |
| E4 | Concept/property | Equations | Multiplication |  | Single digit x multi-digit |  | 2 |  | Arrays | Addition-multiplication | Unknown total |
| E5 | Concept/property | Equations | Multiplication |  | Single digit $x$ single digit |  |  |  |  | Addition-multiplication | Unknown total |
| E6 | Concept/property | Equations | Multiplication |  | Single digit $x$ multi-digit |  | 2 |  |  | Context-equation |  |
| E8 |  | Equations | Multiplication |  | Single digit $x$ multi-digit |  | More than 2 |  |  | Model-equation |  |
| E9 | Concept/property | Equations | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  | Model-equation |  |
| E10 | Concept/property | Equations | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  | Context-equation |  |
| E11 | Concept/property | Equations | Multiplication |  | Single digit $x$ single digit |  | 2 |  |  | Context-equation |  |
| E12 | Concept/property | Equal measures | Division | Partitive | Powers of ten | Single digit |  |  |  | Model-equation |  |
| E13 | Concept/property | Equal measures | Division | Partitive | Powers of ten | Single digit |  |  |  | Model-equation |  |
| E14 | Concept/property | Equations | Division | Partitive |  | Single digit |  |  |  | Context-equation | Group size unknown |
| E16 | Concept/property | Equations | Multiplication and Division |  |  |  |  |  |  | Context-equation | Group size unknown |
| E17 | Concept/property | Equations | Division | Quotative |  | Single digit |  |  |  | Context-equation | Number of groups unknown |
| E18 | Concept/property | Equations | Division |  |  | Single digit |  |  |  | Context-equation |  |
| E20 | Concept/property | Equations | Division |  |  | Single digit |  |  |  | Context-equation |  |
| E23 | Concept/property | Equations | Multiplication |  |  |  |  |  |  | Model-equation |  |
| E25 | Concept/property | Equations | Multiplication |  |  |  |  |  |  |  |  |
| E21 | Concept/property | Equations | Division | Quotative |  | Single digit |  |  |  | Context-equation |  |

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Equations (page 2 of 2)


## Items by CCSSM Clusters (page 1 of 2)

|  | Property | Equal Group | Volume | Equal Measure | Mult. Comparison | Patterns | Rate | Area | Conversions | Equations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Grade 2 - Work with equal groups of objects |  | 69,35,44 |  |  |  |  |  |  |  |  |
| Grade 3 - Represent and solve multiplicative and division problems |  | $\begin{gathered} 1,10,12,14,16,19,30,31,32,33 \\ 4,6,7 \\ \hline \end{gathered}$ |  | 2 |  |  |  |  |  | $\begin{gathered} \hline 1,10,11,12,13 \\ , 14,16,18,2,2 \\ 0,21,23,25,3 \\ 4,5,6,8,9 \end{gathered}$ |
| Grade 3- understand properties and relationships between multiplication and division | $\begin{aligned} & 1,3,4,5,6,9,10,11 \\ & , 12,13,15,48,49, \\ & 50 \end{aligned}$ | 3,36,37,8,15,7 |  | 3 |  |  |  |  |  |  |
| Grade 3 - Solve problems involving the 4 operations and explain patterns in arithmetic |  | $\begin{gathered} 62,15,22,26,38,43,45,46,49,5 \\ 1,52,53,54,57,58,59 \end{gathered}$ |  |  |  | $\left\|\begin{array}{c} 1,11,12,13,14 \\ , 4,5,6 \end{array}\right\|$ |  |  |  | $\begin{array}{c\|} \hline 1,10,11,12,13 \\ , 14,16,18,2,2 \\ 0,21,23,3,4,5, \\ 6,8,9 \\ \hline \end{array}$ |
| Grade 3 - Use place value understanding and properties of operations to perform multi-digit arithmetic |  | $\begin{array}{\|c\|} \hline 62,1,12,14,16,19,22,26,3,30 \\ 31,32,33,38,4,43,45,46,49,51 \\ , 52,53,54,57,58,59,6 \\ \hline \end{array}$ |  | $\begin{gathered} 6,8,11,13,14 \\ 17,18,4 \\ \hline \end{gathered}$ | 20,21,22 |  |  |  |  |  |
| Grade 3 - Solve problems involving measurement |  |  |  | 2,3,6,8 |  |  |  |  |  |  |
| Grade 3 - Understand concepts to area and relate area to multiplication and addition |  |  |  |  |  |  |  | $\begin{gathered} 16,10,12,3,4 \\ 5,6 \end{gathered}$ |  |  |
| Grade 3 - Distinguish between area and perimeter |  |  |  |  |  |  |  |  |  |  |
| Grade 4 - Use four operations with whole numbers to solve problems |  | $63,64,65,66,11,13,17,18,2,20$ $, 21,24,25,27,28,29,34,39,40$, $41,42,47,5,50,55,56,60,70,71$ $, 72,73,74,75,9$ |  | 11,13,14,18,4 | $\begin{array}{\|r\|} \hline 16,17,18,19,1 \\ , 11,12,13,15, \\ 2,3,4,5,6,7,8 \\ \hline \end{array}$ |  | $\begin{array}{\|c\|} \hline 1,10,11,12,13 \\ , 14,15,16,17, \\ 18,19,2,20,3, \\ 4,5,6,7,8,9 \\ \hline \end{array}$ |  |  | $\begin{gathered} 17,30,31,32,3 \\ 3,34,36,37 \\ \hline \end{gathered}$ |
| Grade 4-Gain familiarity with factors and multiples | 34, 35, 43, 44 |  |  |  |  |  |  |  |  |  |
| Grade 4- Generate and analyze patterns |  |  |  |  |  | $\begin{gathered} \hline 10,17,3,7,8,9, \\ 1,13,14,16,2, \\ 3 \\ \hline \end{gathered}$ |  |  |  |  |
| Grade 4 - Generalize place value understanding for multidigit whole numbers | 30,31,26,37,45 |  |  |  |  | 15 |  |  |  |  |
| Grade 4 - Use place value understanding and properties of operations to perform multi-digit arithmetic | $\begin{gathered} 19,21,23,24,25,3 \\ 2,33,39,40,41,42 \\ , 52,53 \\ \hline \end{gathered}$ | $63,64,65,66,10,11,13,17,18,2$ <br> $, 20,21,24,25,27,28,29,34,39$ <br> $40,41,42,47,5,50,55,56,60,70$ <br> $, 71,72,73,74,75,9$ |  |  |  |  |  |  |  |  |
| Grade 4 - Solve problems involving measurement (area and perimeter) and conversion from a larger to smaller unit |  |  |  | 17 |  |  | $\begin{array}{\|c\|} \hline 1,10,11,12,13 \\ , 14,15,16,17, \\ 18,19,2,20,3 \\ 4,5,6,7,8,9 \\ \hline \end{array}$ | $\begin{gathered} 1,12,13,14,15 \\ , 17,19,2,20,2 \\ 1,3,4,5,6 \end{gathered}$ | $\begin{gathered} 10,11,12,13,14,15 \\ 16,17,18,19,2,20,2 \\ 1,22,23,24,25,26,2 \\ 7,28,29,3,30,31,32, \\ 4,5,6,7,8,9 \end{gathered}$ |  |

Items by CCSSM Clusters (page 2 of 2)

|  | Property | Equal Group | Volume | Equal Measure | Mult. Comparison | Patterns | Rate | Area | Conversions | Equations |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | m~.... |  |  |  |  |  |  |  |  |
| Grade 5 - Write and interpret numerical expressions |  |  |  |  |  |  |  |  |  | 38,39,40,41 |
| Grade 5-Analyze patterns and relationships |  |  |  |  |  | 16,2,17,7 |  |  |  |  |
| Grade 5- Understand the place value system | $\begin{gathered} 30,31,36,37,45,1 \\ 8,18,21 \end{gathered}$ |  |  |  |  | 15 |  |  |  |  |
| Grade 5 - Perform operations with multi-digit whole number and decimal to hundredths | 46,47 | 67,68,72,73,74,75,67,68 |  | 15,16,4,13,18 |  |  | $\begin{array}{\|c} \hline 1,10,11,12,13 \\ , 14,15,16,17, \\ 18,19,2,20,3 \\ 4,5,6,7,8,9 \\ \hline \end{array}$ |  |  | 33,34,36,37 |
| Grade 5 - Convert like measurement units within a measurement system. |  |  |  |  |  |  |  |  |  <br> $10,11,12,13,14,15$, <br> $16,17,18,19,2,20,2$ <br> $1,22,23,24,25,26,2$ <br> $7,28,29,3,30,31,32$, <br> $4,5,6,7,8,9$ |  |
| Grade 5 - Understand concepts of volume. Relate volume to multiplication and division |  |  | 1,2,3, 4,5,5,6 |  |  |  |  |  |  |  |
| Grade 5-Apply and extend previous understanding of multiplication and division to fractions |  |  |  |  |  |  | 22,23 |  |  |  |
| Grade 6 - Compute fluently with multi-digit numbers and find common factors and multiples | 34, 35, 43, 44 |  |  |  |  |  |  |  |  |  |
| Grade 6 - Solve real world and mathematical problems involving area, surface, and volume |  |  | 10,11,12,13,7,8,9 |  |  |  |  |  |  |  |


[^0]:    Number EHR-0227057) and the US DOE (S366A0200002))

