OGAP Multiplicative Framework

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Depending upon the strength of multiplicative reasoning students may move up and down between multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem situations and problem structures (Kouba & Franklin, 1995; VMP OGAP, 2006).

CONTEXTS

Equal Groups
Equal Measures
Measure Conversions
Multiplicative Comparison
Patterns
Unit Rate
Rectangular Area
Volume

CONCEPT/PROPERTY

Equations
Properties

Problem Structures

TYPES OF ITEMS
Context
Concept/property

LANGUAGE
Natural (e.g., every)
Mathematical (e.g., per)

MULTIPLICATIVE REPRESENTATIONS
Equal groups
Arrays
Area
Open area
Linear

COMPLEXITY OF NUMBERS
Single digit
Multiple digit
Powers of ten
Multiples of powers of ten
Fractions/decimals

NUMBER OF FACTORS
Two factors
More than two factors

TYPES OF DIVISION
Partitive
Quotative

UNDERSTANDING AND USE OF PROPERTIES
Associative
Commutative
Distributive
Equality
Identity
Multiplicative Inverse

UNDERSTANDING AND USE OF RELATIONSHIPS
Addition - Multiplication
Doubling and Halving
Model - Equation
Multiples and Factors
Meanings of Remainders
Context - Equation
Context - Model
Powers of ten

Problem Situations
Refers to known and unknown information in a problem. For example, in equal group problems there might be an unknown product, or an unknown number of equal groups, or an unknown quantity in a group. See page 4 for some examples. Also see the CCSSM page 89.

About OGAP Frameworks

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems. There are two major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions: 1) Problem contexts and structures, and 2) OGAP Multiplication and Division Progressions.

This page identifies problem contexts and problem structures for multiplication and division problems. Pages 2 and 3, the OGAP Multiplication and Division Progressions, are tools to help teachers analyze evidence of student thinking and make instructional decisions. Page 4 has samples of different problem contexts.

For students to become strong multiplicative reasoners they must interact with a range of problem contexts, contextual situations, and problem structures. The CCSSM specifically identifies problem contexts at targeted grades on a progression from equal groups and measures, and area situations at grades 2 and 3 to measurement conversions, area, volume, multiplicative patterns, and multiplicative comparison problem situations at grades 4 and 5, and unit rates at grade 6. This progression, among other things, is designed to prepare students to engage in proportional situations using multiplicative reasoning.

Consistent with the CCSSM, the OGAP Progressions on pages 2 and 3 show the development of understanding and strategies from the link between repeated addition and multiplication in an equal groups model to the development of efficient and generalizable multiplicative strategies through the open area model, and understanding of place value, properties of operations, and relationships.

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) which was funded by NSF (EHR-0227057) and the US DOE (S366A020002).

OGAP Multiplication Progression

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problems structures and contexts.

Unitizes into equal groups. uses visual models, understanding of place value, and properties of operations

Multiplicative

Multiplicative Strategies

Algorithms

Partial Products

$$\begin{array}{r} 16 \\ \times\ 42 \\ \hline 12 \\ 20 \\ 240 \\ \hline 400 \\ 672 \end{array}$$

Traditional

$$\begin{array}{r} {\scriptstyle 21} \\ 16 \\ \times\ 42 \\ \hline 32 \\ 640 \\ \hline 672 \end{array}$$

Distributive Property

$4 \times 16 = 4(10 + 6)$
$= 4(10) + 4(6)$
$= 40 + 24$
$= 64$

Associative Property

$(8 \times 2) \times 5 = 8(2 \times 5)$
$= 8 \times 10$
$= 80$

Doubling & Halving

$16 \times 4 = 8 \times 8$
$= 64$

Known or Derived Fact

$4 \times 6 = 24$

Commutative Property

$16 \times 4 = 4 \times 16$

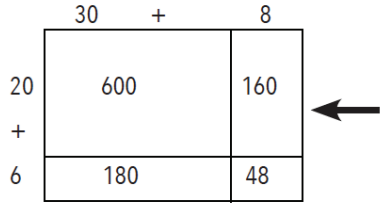
Powers of Ten

$5 \times 400 = 5 \times 4 \times 10 \times 10$

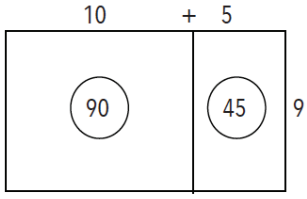
Transitional

Transitional Strategies

Open Area Model *Considers both dimensions of an array or area model*

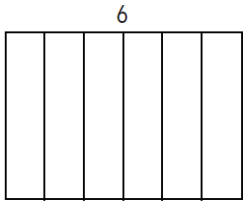


38 x 26 = 988



15 x 9 = 135

Area Model



6 x 4 = 24

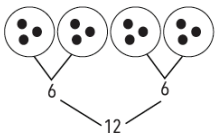
Considers BOTH dimensions of an array or area model, moving away from needing to see every square unit

EarlyTransitional Strategies

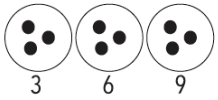
Skip Counting 3, 6, 9, 12, 15

Building up

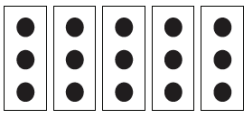
3+3+3+3
6+6
12



Skip Counting with a Model

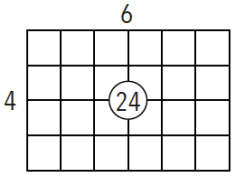


Equal groups in an array



Considers only ONE dimension of an array or area model

Area Model – 6 x 4 = 24



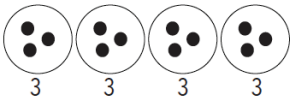
Considers BOTH dimensions of an array or area model

Additive

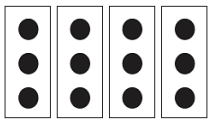
Additive Strategies

Repeated addition with or without a model – 3 x 4=12

$3 + 3 + 3 + 3 = 12$



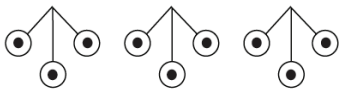
Subitizing in small groups



3 + 3 + 3 + 3

Early Additive Strategies

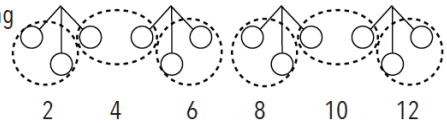
Modeling, counting by ones



Inconsistent Grouping

Modeling, counting by subgroups

3 x 4=12



Non-Multiplicative Strategies

Adds or subtracts factors
Models factors incorrectly
Guesses
Uses incorrect operation
Not enough information
Uses procedures incorrectly

Underlying Issues and Errors

- Doesn't consider reasonableness of solution
- Error in: calculation, place value, vocabulary, property or relationship, equation, or model

Units inconsistent or missing

OGAP Division Progression

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problems structures and contexts.

Unitizes into equal groups, uses visual models, understanding of place value, and properties of operations

Multiplicative Strategies

Algorithms

Partial Quotients

17 | 585
340 x 20
245
170 x 10
75
68 x 4
7 34 r 7

4
10
20 34 r 7
17 | 585
340
245
170
75
68
7

Traditional

34 r 7
17 | 585
51↓
75
68
7

Distributive Property

$35 \div 7 = (21 + 14) \div 7 = 3 + 2 = 5$

Treats the remainder appropriately given problem situation

Inverse relationship between multiplication and division

$35 \div 7 = 5 \quad 7 \times ? = 35$

Known or Derived Fact

$21 \div 7 = 3$

Transitional Strategies

Inefficient partial quotients

61 | 756 — 12 R 24
183 x 3
573
183 x 3
390
183 x 3
207
183 x 3
24

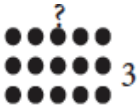
Trial and error to find a quotient

61 | 756
732
24
12 R 24

2 x 61 = 122
20 x 61 = 1220
4 x 61 = 244
8 x 61 = 488
15 x 61 = 915
10 x 61 = 610
12 x 61 = 732

EarlyTransitional Strategies

Models in an array to find missing dimensions



$15 \div 3 = ? \quad 3 \times ? = 15$

Skip Counts to find the number of "skips" with and without a model

$15 \div 3 = 5$ 3, 6, 9, 12, 15 (5 skips)

Additive Strategies

Repeated subtraction or adding up to

61 | 350 — 5 R 45
61
289
61
228
61
167
61
106
61
45

61
+61
122
+61
183
+61
244
+61
305
+61
366

350
-305
r 45

Sharing out in equal groups as repeated subtraction or addition

There are 8 cookies to share equally with 4 children. How many cookies does each child get?

8 cookies ÷ 4 children = 2 cookies per child

Represents the 4 children with circles and then fills them equally

Twenty-four cookies were put into bags of 4 cookies each. How many bags were filled?

24 cookies ÷ 4 cookies per bag = 6 bags

Pulls out 4 cookies at a time until 24 cookies are used

Early Additive Strategies

Sharing out by ones

Sharing out randomly by subsets

Sharing 4, then 2, then 5, and so on

Non-Multiplicative Strategies

Adds or subtracts dividends/divisors
Models problem incorrectly
Guesses
Uses procedures incorrectly
Uses incorrect operation
Not enough information

Underlying Issues and Errors

- Doesn't consider reasonableness of solution
- Error in: calculation, place value, vocabulary, property or relationship, equation, or model
- Misinterprets the remainders
- Units inconsistent or missing

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OGAP Sample Problem Contexts

Important: The examples below do not represent the full range of each problem context. * problem situations

Multiplication Examples	Division Examples
Equal Groups * *Unknown product example* Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there altogether? 12 boxes × 8 crayons = n crayons	* *Unknown number in each group example* Mark had a box of 64 crayons. He shared the crayons equally with 4 people. How many crayons did each person get? 64 crayons ÷ 4 people = *n* crayons per box
Equal Measures * *Unknown product example* It takes 14 inches of ribbon to make one bow. How many inches of ribbon will it take to make 7 bows? 7 bows × 14 inches per bow = n inches	* *Unknown number of groups example* Sam has 15 yards of material. He is making a design that needs 3 yards per design. How many designs can Sam make? 15 yards ÷ 3 yards per design = *n* designs
Unit Rates * *Unknown product example* Sam works at the grocery store. He is paid $7.00 per hour. He worked 22 hours last week. How much money did Sam earn last week? 22 hours × $7.00/hour = *n* dollars	* *Unknown rate example* Sam earned $154.00 last week. He worked 22 hours. How much did Sam earn per hour? $154.00 ÷ 22 hours = *n* $/hour
Multiplicative Comparison * *Larger unknown example* The students in Mrs. Gilbert's class planted bean and corn seeds. The bean plants grow 3 times faster than the corn plants. When the corn plants measure 2 inches, how tall will the bean plants be? 2 inches × 3 = *n* height of bean plants	* *Multiplier unknown example* Bill's garden is 240 square feet. Leslie's garden is 20 square feet. How many times greater is Bill's garden than Leslie's garden? 240 square feet ÷ 20 square feet = *n* times bigger
Patterns **Larger unknown example* Tammy is decorating tables with vases of flowers for a party. She used the chart to the right to keep track of how many flowers she needed. Based on the information in the table, how many flowers does she need to fill 15 vases? 15 vases of flowers × 4 = *n* flowers	<table><tr><th>Number of Vases of Flowers</th><th>Number of Flowers</th></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>12</td></tr><tr><td>4</td><td>16</td></tr><tr><td>15</td><td></td></tr></table>
Measurement Conversion * *Unknown product example* Tammy is 5 feet tall. How many inches tall is Tammy? 5 feet × 12 inches/foot = *n* inches	* *Unknown quotient example* Tammy is 60 inches tall. How many feet tall is Tammy? 60 inches ÷ 12 inches/foot = *n* feet
Area * *Unknown product example* Linda's kitchen floor measures 12 feet by 7 feet. What is the area of Linda's kitchen floor? 12 feet × 7 feet = *n* square feet	* *Unknown factor example* Linda's kitchen floor is 150 square feet. The length of one dimension is 10 feet. What is the length of the other dimension of the kitchen floor? 150 square feet ÷ 10 feet = *n* feet
Volume * *Unknown product example* The dimensions of a fish tank are 10 inches by 12 inches by 18 inches. If you filled the tank to the top, how much water would be in the tank? (10 inches × 12 inches × 18 inches = *n* cubic inches)	* *Unknown factor example* A rectangular container holds 250 cubic inches of liquid. the base of the container is 10 inches and the depth 5 inches. What is the height of the container? 250 cubic inches ÷ (10 inches × 5 inches) = *n* inches
OGAP Equation Example: Write a story problem that goes with the equation 6 × 5 = 30.	**OGAP Property Example**: Ann knows the answer to 9 × 5. Explain how can she use this information to solve 45 ÷ 9?

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