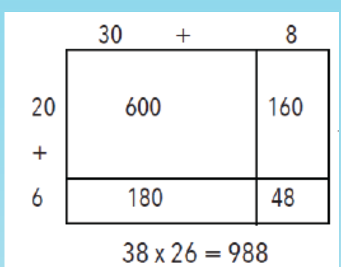
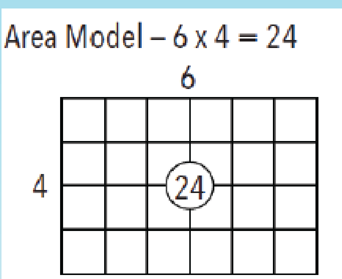
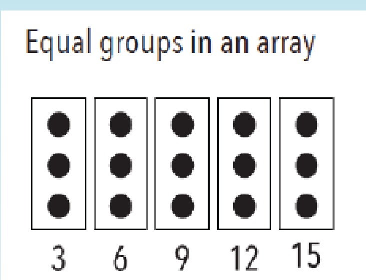
OGAP

ONGOING ASSESSMENT PROJECT

Multiplicative Reasoning







Session 0: OGAP Overview
Session 1: What is Multiplicative Reasoning?
Session 2: Big Ideas
Session 3: Properties of Operations
Session 4: Structures of Problems
Session 4.2: Interpreting and Solving Word Problems
Session 5: Meaning of the Quantities
Session 6: Understanding Algorithms
Session 7: Additive to Multiplicative
Session 7.2: Subitizing
Session 8: Using OGAP Pre-assessments
Session 9: Developing Math Fact Fluency
Session 10: Division
Session 11: Navigating the Item Bank

OGAPMath LLC

June 2017

Personal Parking Lot

Questions, Big Ideas, and Other Thoughts

Session 1 Stop and Jot

Reading 1: Page 1 to Number Relationships and Actions in Addition
Reading 2: Number Relationships and Actions in Addition to top of page 4
Reading 3: Page 4 to Table 1.1 (page 5)
Reading 4: Absolute and Relative Difference (page 5) to Multiplicative Reasoning from a Teaching and Learning Perspective

Part I: What questions related to multiplication might you ask your students about this picture?

Muffin Tin Figure on Page 40 – Fosnot, K. (2001). CD – *Young Mathematicians at Work – Constructing Multiplication and Division*, Heinemann, NH.

Part II: Examine the student work in figures 1.6-1.9 and read the quote from *NCTM Principles and Standards for School Mathematics* (PSSM, 2000). Based on your experience, thinking, and the information mentioned, complete the statement in the box below.

"Multiplicative reasoning is more than just doing multiplication or division. It is about understanding situations in which multiplication or division is an appropriate operation. It involves a way of viewing situations and thinking about them." (PSSM, 2000)

The student who has multiplicative reasoning demonstrates…

Part III: Watch the video clips and respond to the questions below.

Watch Introduction to Muffin Tins

Clip 1: Take note of questions that Dana asks and her strategies for facilitating students' discourse.

a) What did the teacher do to facilitate the conversation and learning so that students engaged in discourse towards acquiring MR?

b) What strategies do the students use that are found on the OGAP Multiplicative Reasoning Framework? What are instructional implications based on the location on the OGAP framework?

c) What mathematics do you think she will try to get with the next set of trays?

d) What is the teacher doing to facilitate a focus on composite units?

Clip 2

a) We have already talked about what the teacher did to facilitate student learning. Are there more observations from this clip?

b) Identify some strategies that students used found on the OGAP Multiplicative Reasoning Framework. What are instructional implications based on the location on the OGAP framework?

c) What mathematics do you think she will try to get with the next set of trays?

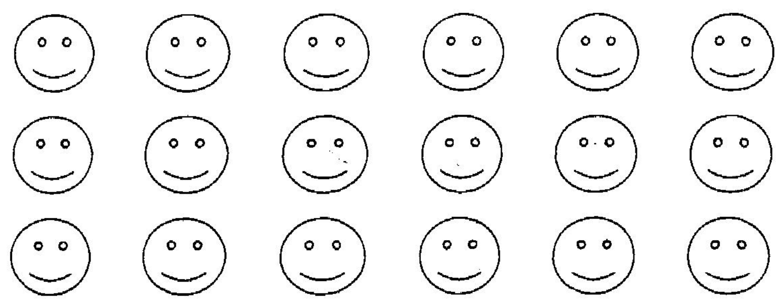
d) What evidence is there that students are struggling with composite units?

Part IV: Assume that you did the Muffin Tin problem with the students in your classroom.

1) What aspects of the question below make it a good choice as a follow---up to the lesson? What concepts were considered when the item was designed?

Write an equation to match this picture.

Explain your thinking.



2) With a partner sort the student work in the different strategies that students used (regardless of the correctness of the solution).

3) Identify the location of the different strategies on the OGAP Multiplicative Reasoning Framework. Select one piece of work and indicate possible instructional implications based on the evidence and location of the framework.

2a: The Big Ideas and Properties in Multiplication

Big Idea Poster Development (in groups)

1) Study the prompt you are assigned.
2) Read the sections of the book that are related to the prompt.
3) Discuss and capture your best thinking *in response to your prompt* on a piece of chart paper:
    a) Include a visual model(s) in your response with accompanying explanations and/or notation so that the poster can stand-alone for others to interpret and understand the concepts underlying your prompt.

PROMPT 1 - Unitizing

Your Task: Provide examples and counter examples of unitizing. Explain what unitizing is and why is it critical to students' acquisition of multiplicative reasoning. Give some specific examples to illustrate unitizing. Where else in mathematics do we need unitizing?
(Chapter 4 - pages 56 -58)

PROMPT 2 – Using Arrays Effectively

Your Task: Compare and contrast equal groups and array models. Give specific examples of the benefits of using arrays over equal groups models. Explain why arrays can be a limiting model; be sure to consider the stages students may move through as they develop flexibility with arrays.
(Chapter 3 - pages 43 - 45)

PROMPT 3 – Area and Open Area Models

Your Task: Compare and contrast area and open area models. Illustrate specific examples of how each can be used to help develop multiplication concepts and properties. Provide examples as to why open area models are referred to as the most flexible model.
(**Chapter 3** - **pages 45 - 47)**

PROMPT 4 – Subitizing and Quick Images

Your Task: Distinguish between conceptual and perceptual subitizing and the role that conceptual subitizing plays in development of multiplicative concepts? Give specific examples of how subitizing can be used to develop some of the big ideas in multiplication and division.
(Chapter 3 - pages 47 - 51)

PROMPT 5 – Place Value and Powers of 10

Your Task: Illustrate why multiplying by 10 is not just adding a 0 (a trick). Provide examples of multiplying by powers of ten (e.g., 5 x 6 to 50 x 6 to 50 x 60 that show the relationship between symbolic representation, visual models, and written and oral language that helps bring understanding to multiplying by powers of ten. **(Chapter 4 - pages 60 - 63)**

PROMPT 6 – Commutative Property

Your task: Using visual models, illustrate the commutative property for multiplication. Consider the different models (equal groups, arrays, and area models) for multiplication. Illustrate the benefits of using one model over another to build understanding of the commutative property. Provide examples where the use of the commutative property can provide students flexibility when solving problems. **(Chapter 4 - pages 71 - 73)**

PROMPT 7 – The Distributive Property, Visual Models, and Multiplication Facts

Your Task: Make a list of 3 multiplication math facts that you think students have the most difficulty learning. For each of these, illustrate the use of the distributive property and the open area model to help students derive difficult multiplication math facts. Give more than one example for each of the math facts.
(Chapter 9 - pages 181-183)

PROMPT 8 – Associative Property

The CCSSM at grade 5 specifically requires students to make the connection between visual models and the associative property.

> CCSS.MATH.CONTENT.5.MD.C.5.A (HTTP://WWW.CORESTANDARDS.ORG/MATH/CONTENT/5/MD/C/5/A/)
> Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

Your Task: Using rectangular prisms and examples illustrate the associative property. In particular, provide multiple examples of how understanding of the associative property and its link to volume of rectangular prisms can help develop flexibility when multiplying numbers.
(Chapter 4 - pages 73 - 75)

2b – Big Ideas Notes

Unitizing	**Using Arrays**
Area and Open Area Models	**Subitizing**

Place Value and Powers of Ten	Commutative Property
Distributive Property and Multiplication Facts	**Associative Property**

1) The table below shows the cost of lollipops at Brian's Candy Store.

Number of Lollypops	Cost
3	$.15

1) The table below shows the cost of lollipops at Brian's Candy Store.

Number of Lollypops	Cost
3	$.15
4	$.20
5	$.25
6	$.30
7	$.35
8	$.40

A. What is the cost of 9 lollipops at Brian's Candy Store? Show your work.

B. Write a rule to find out the cost of 10 lollipops at Brian's Candy Store.

2) Abby has 8 quarts of ginger ale. She plans to use it all to make the punch recipe below. How many gallons of juice does Abby need to buy?

Fruit Punch Recipe

3 quarts juice
1 quart ginger ale
4 scoops sherbet

4 quarts = 1 gallon

3) Mr. Jones ordered office supplies.

He ordered 7 cases of paper. There are 10 packages of paper per case. Each package contains 500 sheets of paper. How many sheets of paper did he order?

Show your work.

4) Ms. Smith's class keeps track of the number of pages they read each week. Below is a chart with some missing information.

Students	**Pages Read**
Mike	54
Tina	
Luke	39
Cara	27
Seth	

Seth read 4 times more pages than Cara. How many pages did Seth read?

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) funded by NSF EHRo 0227057 and the US DOE (S366A20002).

5) It takes 14 inches of ribbon to make one bow. How many inches of ribbon will it take to make 7 bows?

6) The price of gas in 1960 was $.19 per gallon. Today the price is 14 times more than in 1960. What is the price of gas per gallon today?

7) How many millimeters are in $5\frac{1}{2}$ centimeters?

10 millimeters = 1 centimeter

8) During migration, humpback whales swim about 6 miles per hour. After 25 hours, about how far have they traveled?

9)

A. Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there all together? Show your work.

B. John bought 12 boxes of crayons. Each box contained 64 crayons. How many crayons were there all together? Show your work.

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP) funded by NSF EHR-0227057 and the US DOE (S366A20002).

Step 1: With a partner sort the problems into three categories – 1) Easiest; 2) Moderate difficulty; and, 3) Most challenging

Step 2: Make notes of **features of the problems** that would make them more or less challenging for students.

Easiest	Moderate	Most Challenging
List Problems:	List Problems:	List Problems:
Features of problems:	Features of problems:	Features of problems:

4e- The Visual Model Case Study Handout

Ms. Ward is a fourth grade teacher. It is the beginning of the school year and she is getting ready to start her first unit on multiplication. The unit focuses on having students use area models to make sense of a variety of important math concepts such as properties of operations and basic fact fluency. Before beginning the unit she decides to give her students the following task.

> Look at this equation.
>
> $\boxed{8 \times 4 = 32}$
>
> Draw a visual model that represents this equation.

Ms. Ward knows that asking them to draw a model when there is no context will most likely reveal their default model (the model they are most comfortable using when not influenced by context).

Sorting the Student Work

Sort the student work into piles based on the type of model students drew. Record the information in the chart below.

No Model	Equal Groups	Arrays	Area	Open Area

<table>
<tr><th colspan="3">Concept/Property Item Analysis</th></tr>
<tr><th>Item</th><th>Does not apply or demonstrate understanding of targeted concept, relationship, or property</th><th>Demonstrates understanding of targeted concept, relationship, or property</th></tr>
<tr><td>8 x 4 = 32

Draws a visual model.</td><td></td><td>Equal Groups:

Arrays:

Area:

Open area:

Other:</td></tr>
<tr><td colspan="3">Underlying issues or concerns in student solutions:
______ Unreasonable   ______ Property or relationship error
______ Misinterprets remainders   ______ Calculation error
______Place value error   ______ Equation error
______ Units inconsistent or absent   ______ Model error
______ Other   ______ Vocabulary error</td></tr>
<tr><td colspan="3">Notes:</td></tr>
</table>

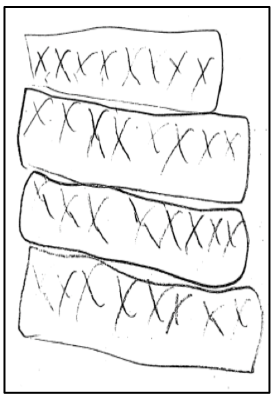
4e- The Visual Model Case Study Handout

1) What is the evidence of developing understanding?

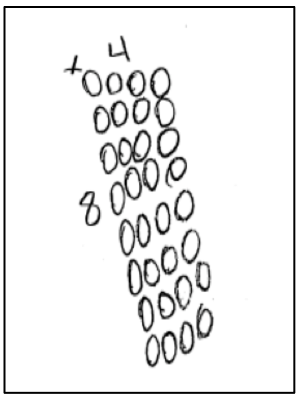
2) What issues are evidenced in the student work?

Ms. Ward decided she would select and sequence student work from the sort to help students understand area models before beginning the unit.

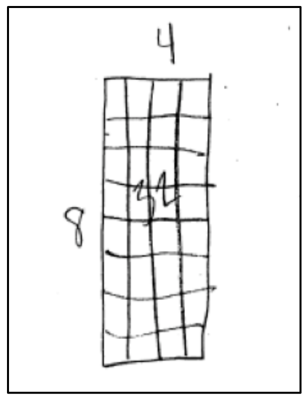
Ms. Ward's Selection



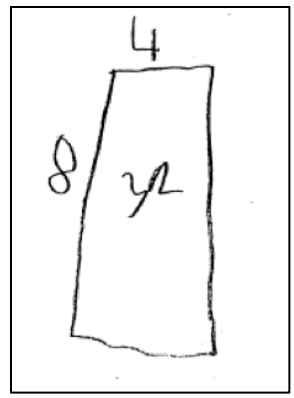
Student G



Student F



Student L



Student J

Engaging Students in Solutions

a) Why do you think Ms. Ward chose these 4 solutions?

b) Why do you think Ms. Ward chose to sequence them in this order?

c) What questions would you ask students about the solutions to help them move towards understanding an area model?

The student work below shows four different ways that a student may not be interpreting the meaning of the quantities in the problem or in the solution. Review the student work and then answer the following questions for each response.

Student Response A

Jo rides her bike 13 miles per hour.

How long will it take her to ride 52 miles?
Show your work.

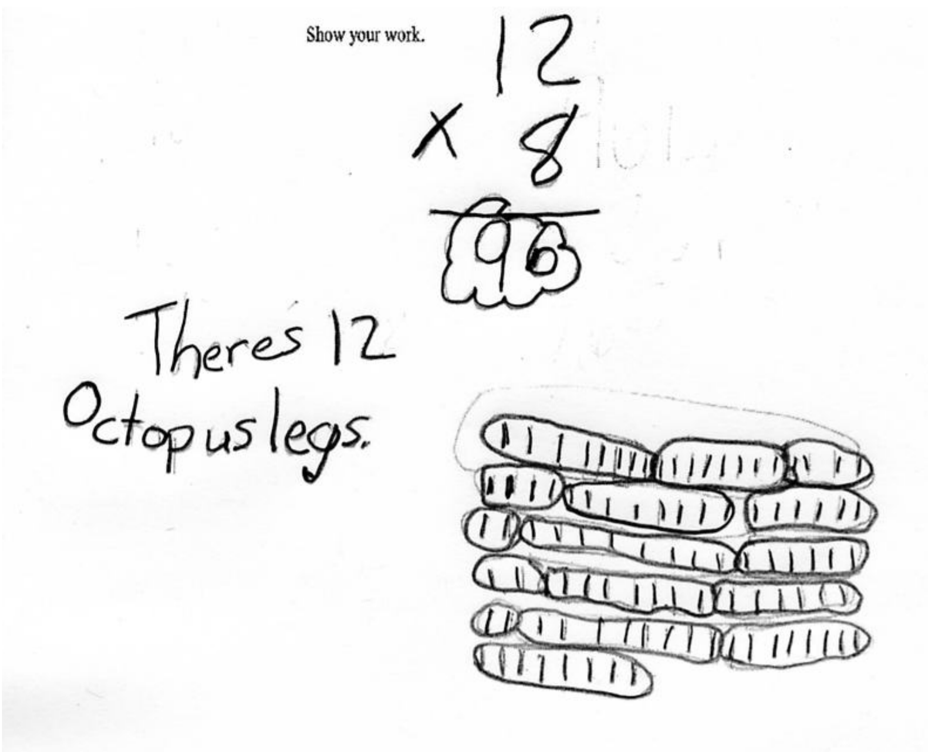
$$\begin{array}{r} 13 \\ +\ 52 \\ \hline 65 \end{array}$$ miles

1) What is the evidence that the student may not be interpreting the meaning of the quantities in the problem?

2) Suggest some questions you might ask the student you might do to help the student understand the meaning of the quantities in the problem and the solution.

Student Response B

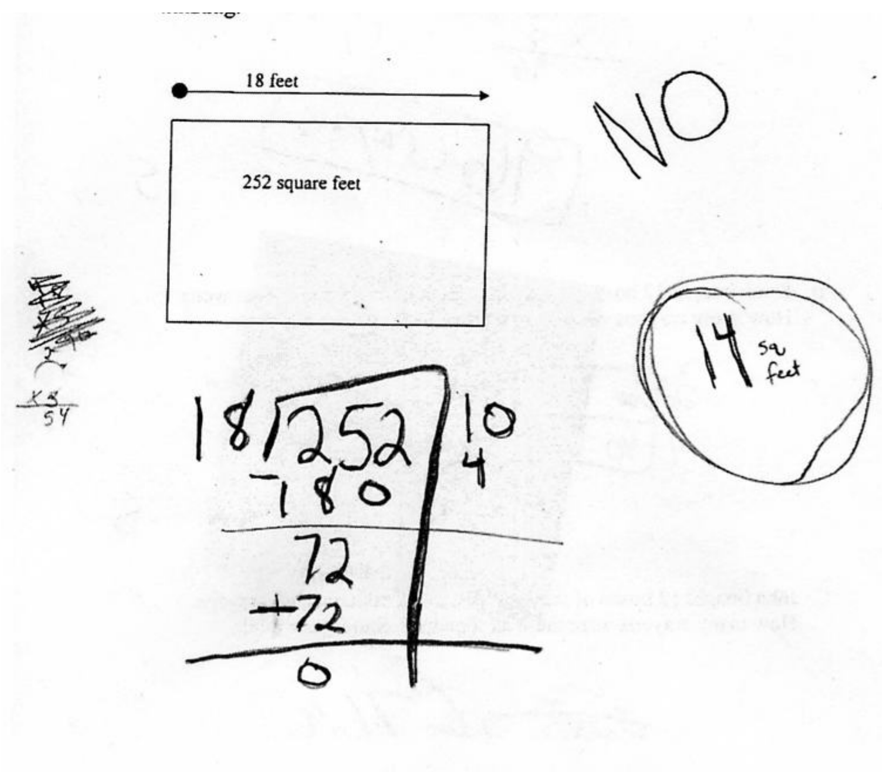
There are 96 octopus legs at the aquarium. An octopus has eight legs. How many octopuses are there?



1) What is the evidence that the student may not be interpreting the meaning of the quantities in the problem?
2) Suggest some questions you might ask the student or activities you might do to help the student understand the meaning of the quantities in the problem and the solution.

Student Response C

Dean puts a fence around his garden. The area of his garden is 252 square feet. One side of his garden is 18 feet long. Dean has 70 feet of fencing. Does Dean have enough fencing to go around his garden? Explain your thinking.



1) What is the evidence that the student may not be interpreting the meaning of the quantities in the problem?

2) Suggest some questions you might ask the student or activities you might do to help the student understand the meaning of the quantities in the problem and the solution.

Student Response D

The Dilly Market has 4 shelves of pickles.

Each shelf has 5 jars.
Each jar has 6 pickles.

How many pickles are there in all?

$4 \times 5 \times 6$ is 120

1) What is the evidence that the student may not be interpreting the meaning of the quantities in the problem?
2) Suggest some questions you might ask the student or activities you might do to help the student understand the meaning of the quantities in the problem and the solution.

1) **Individually** solve the following problem using an open area model, partial products, and the traditional algorithm.

$27 \times 38 =$

With your table group answer questions 2 and 3 by sketching the three solutions on chart paper and showing the relationships.

2) Explain how the **partial products algorithm** is related to the **area model**.

3) Explain how the **traditional algorithm** is related to the area model.

4) Explain how the open area model is related to **multiplying algebraic expressions** (e.g., $(2x + 3)(3x + 4) =$).

Session 7:
The Bridge from Additive to Multiplicative Strategies

Why is it important to know about the strategies students use in multiplicative situations?

1. Attain procedural fluency
2. Inform instruction

Why is it important to know about the strategies students use in multiplicative situations?

Attain procedural fluency

Procedural fluency "refers to the knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently."

(Adding It Up! NRC (2000))

Inform instruction

Understanding students' strategies helps teachers move students towards more efficient strategies.

Multiplicative Reasoning

- Additive Strategies
- Transitional Strategies
- Multiplicative Strategies

Multiplicative Reasoning

Additive Strategies

Transitional Strategies

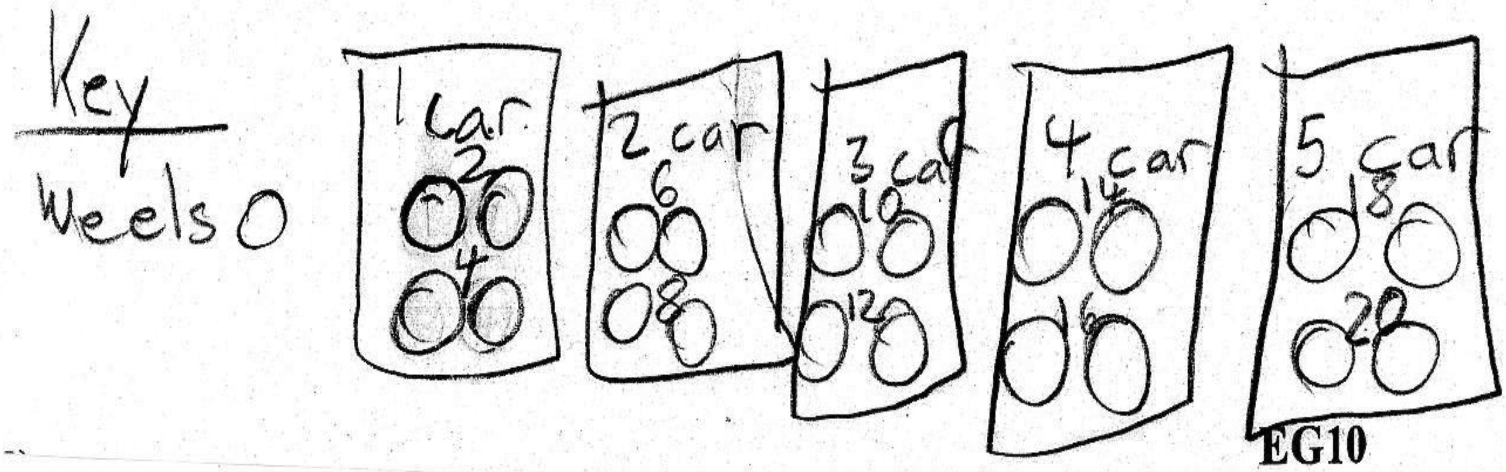
Multiplicative Strategies

- Modeling – counting by ones
- Modeling – counting by subgroups
- Repeated addition

- **Review the student work on pages 23 – 25.**
- **Identify the additive strategy that is evidenced in the student work.**
- **Using the OGAP Progression discuss – What might be the next instructional step for this student?**

There are five cars in the parking lot. Each car has four wheels. How many wheels are there in all? 20 weels

Show your work.



Ryan competed in the long jump in a track meet.

On his first attempt he jumped 2.5 meters.

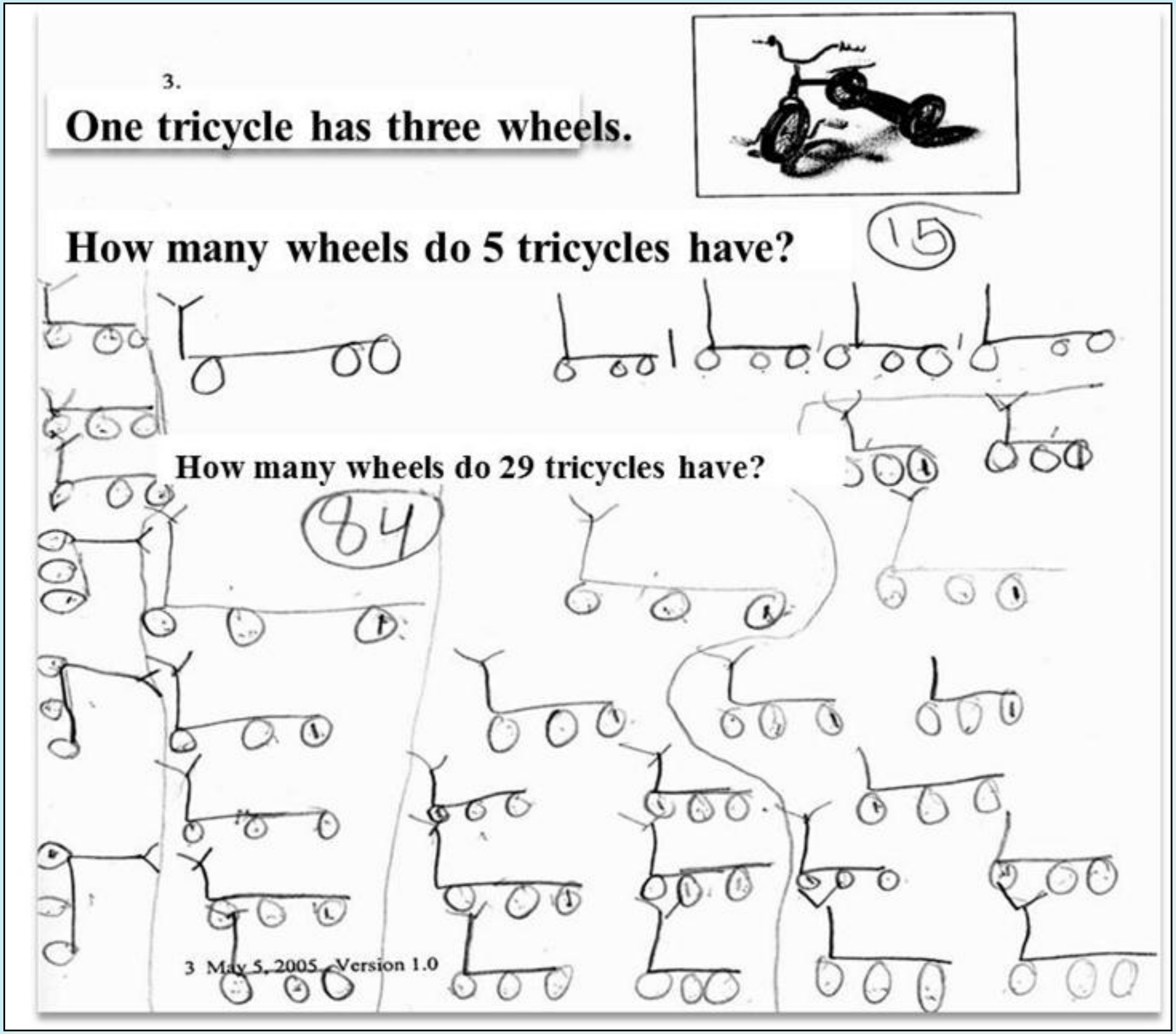
On his second attempt he jumped 2.0 meters.

How many centimeters did Ryan jump in all?.

1 meter = 100 cm

$$\begin{array}{r} 2.5 \\ +\,2.0 \\ \hline 4.5 \end{array}$$

So he jumped 4.5 meters total. And I know that 1 meter = 100 cm so 100+100+100+100 +50 = 450 cm


3.
One tricycle has three wheels.
How many wheels do 5 tricycles have?
15
How many wheels do 29 tricycles have?
84
3 May 5, 2005 Version 1.0

Multiplicative Reasoning

Additive Strategies

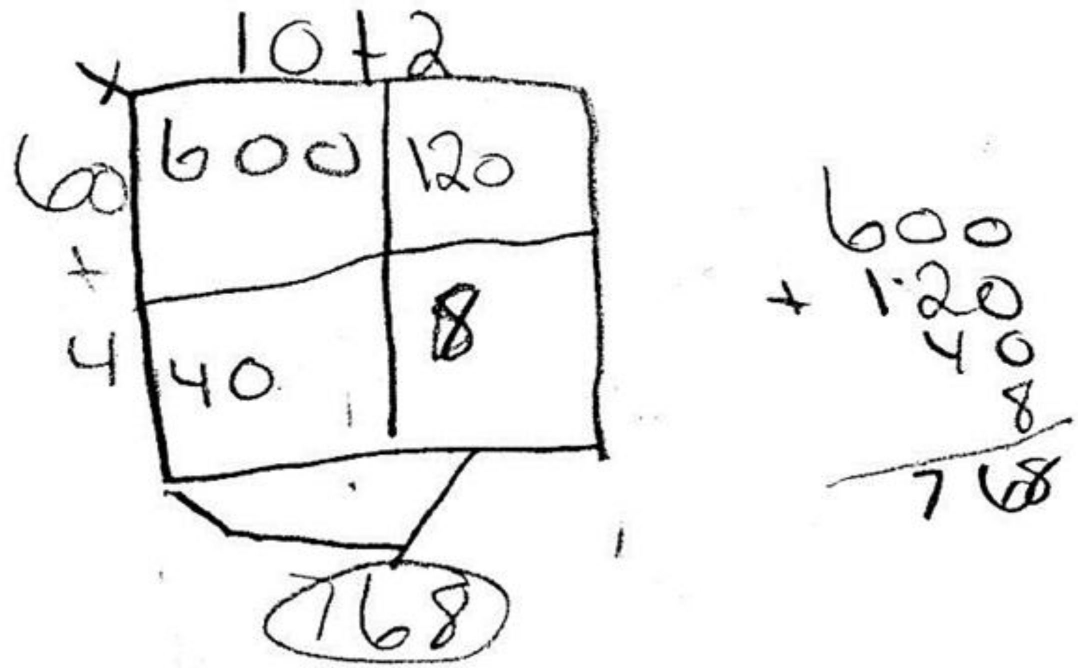
Transitional Strategies

Multiplicative Strategies

- **Review the student work on pages 27-29.**
- **Identify the transitional strategy that is evidenced in the student work.**
- **Using the OGAP Progression discuss – What might be the next instructional step for this student?**

- Skip counting
- Doubling/Building up or down
- Using models

. John bought 12 boxes of crayons. Each box contained 64 crayons.
How many crayons were there all together? Show your work.



3.

One tricycle has three wheels.

How many wheels do 5 tricycles have? ork.

3, 6, 9, 12, 15 18

answer

15

How many wheels do 29 tricycles have?

3, 6, 9, 12, 15, 18, 21, 24, 27
30, 33, 36, 39, 42, 45, 48, 51
54, 57, 60, 63, 66, 69, 72, 75
78, 81, 84, 87

answer

87

There are five cars in the parking lot.
Each car has four wheels. How many wheels are there is all?

8 + 8 = 16 4

$$\begin{array}{r} 16 \\ +\ 4 \\ \hline 20 \end{array}$$

4 X 5 = 20

20 weels in each

Multiplicative Reasoning

Additive Strategies	Transitional Strategies	Multiplicative Strategies

- **Review the student work on pages 31 – 34.**
- **Identify the additive strategy that is evidenced in the student work.**
- **For each problem and response identify how the structure of the problem provided the opportunity to use the strategy.**
- **What might you do to "push" on the strength of the multiplicative strategy strategy illustrated in the student work?**

- Known or derived facts
- Algorithms
- Distributive Property
- Associative Property
- Commutative Property
- Multiples of ten
- Doubling and halving
- Patterns (functional)

Rod bought 13 tins of mints. Each tin contained 27 mints. How many mints were there all together?

Show your work.

$$\begin{array}{r} 27 \\ \times\ 13 \\ \hline 200 \\ 70 \\ 60 \\ 21 \\ \hline 351\cancel{0} \end{array}$$

Mr. Jones is ordering office supplies. He has ordered 7 cases of paper. There are 10 packages of paper per case. Each packages contains 500 Sheets of paper. How many sheets of paper did he order?

35,000 sheets

$$\begin{array}{r} 10 \\ \times\ 7 \\ \hline 70 \end{array} \qquad \begin{array}{r} 500 \\ \times\ 70 \\ \hline 35{,}000 \end{array}$$

A class has set a goal that each student will read 45 pages this week. There are 16 students in the class. How many pages will they have read altogether by the end of the week?

$45 \times 16 = ?$

$90 \times 8 = 720$

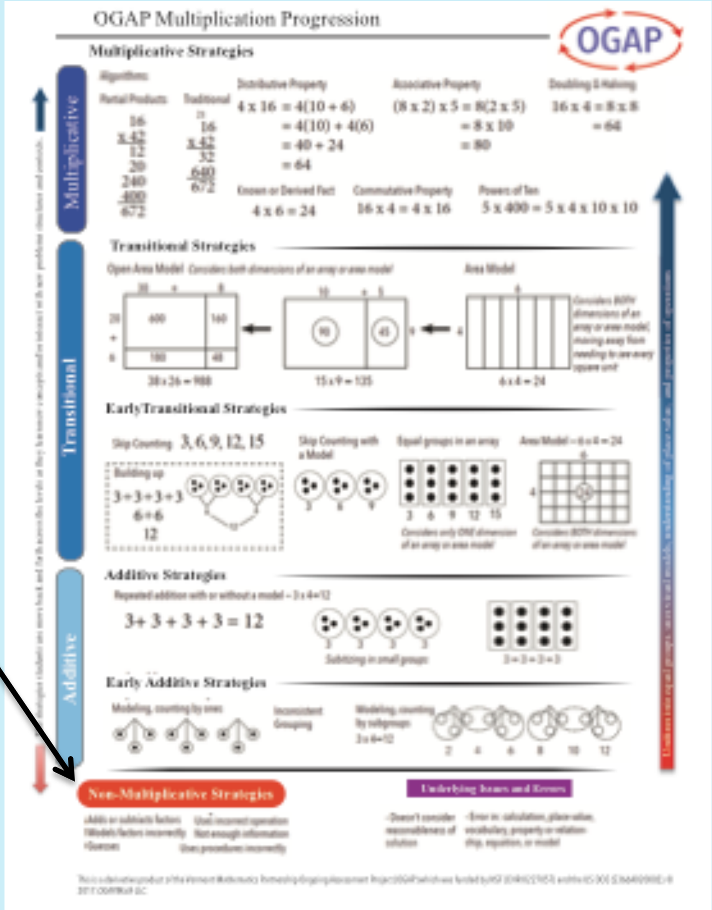
A. Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there all together? Show your work.

$$\begin{array}{r} \scriptstyle 1 \\ 12 \\ \times\ 8 \\ \hline 96 \end{array}$$ crayons

As students begin developing multiplicative strategies or encounter unfamiliar contexts or more complex number, they may revert to additive strategies and non-multiplicative reasoning. *(Kouba and Franklin, 1995)*

Evidenced by…

- Adding or subtracting factors
- Randomly applying numbers, operations, or strategies
- Modeling factors incorrectly
- Using a procedure incorrectly



Evidence of Non-Multiplicative Reasoning

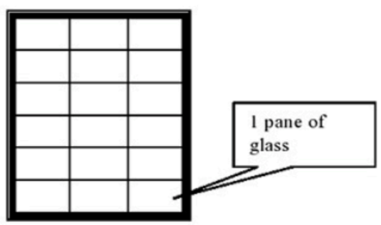
Sarah bought 12 boxes of crayons. Each box contained 24 crayons. How many crayons were there all together? Show your work.

$$\begin{array}{r} 12 \\ +24 \\ \hline 34 \end{array}$$

Case Study

Lesson Goal: To solve multiplication problems with 3 single-digit factors in a variety of contexts.

The Smith's have 3 windows on the front side of their house like the picture below. Each window contains many panes of glass.



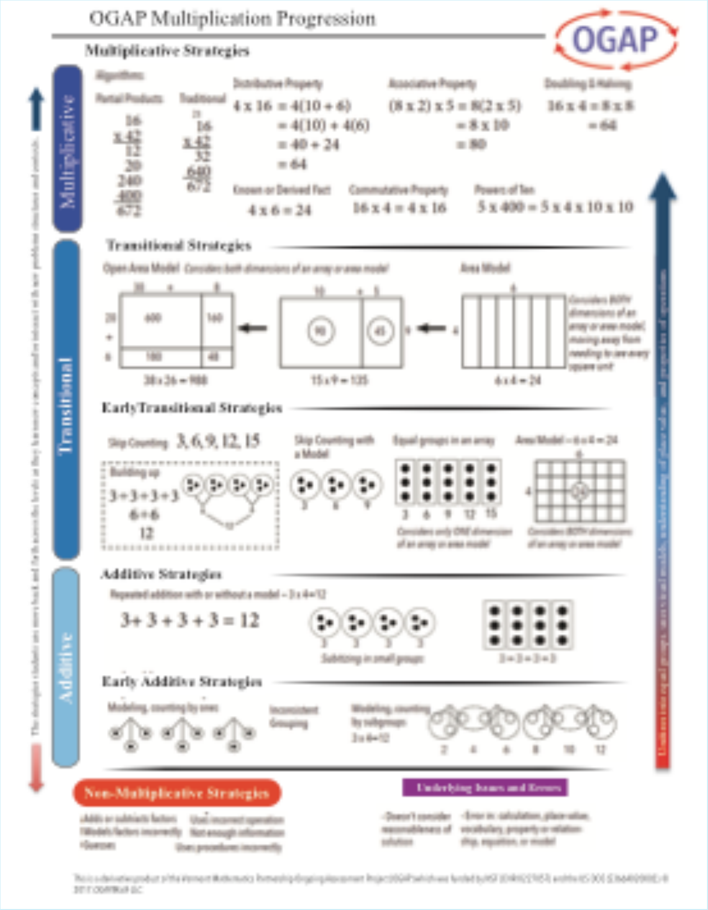
How many panes of glass are there in the 3 windows on the front of the Smith's house?

Show your work.

1. Solve the task.
2. What errors might students make?
3. Why do you think Ms. Ward selected this task?

Examining Student Work

Sort



After Sorting, record the evidence

The Ongoing Assessment Project

Questions to ask when analyzing student work:

» What is the evidence of developing understanding that can be built upon?

» What are issues or concerns that are evidenced in student work?

Select and sequence the student work.

a) Select and sequence student work from the solutions you sorted to respond to the evidence in the student work.

b) Why did you choose these solutions?

c) Why did you sequence them in the order you did?

d) What questions would you ask students about the solutions to help move them along the multiplication progression?

Be prepared to share with another group.

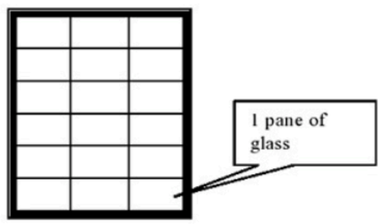
7a- The Smith's Windows Case Study Handout

Lesson Goal: To solve multiplication problems with 3 single-digit factors in a variety of contexts.

Exit Card

1) Solve the task.
2) What errors might students make?
3) Why do you think Ms. Ward selected this task?

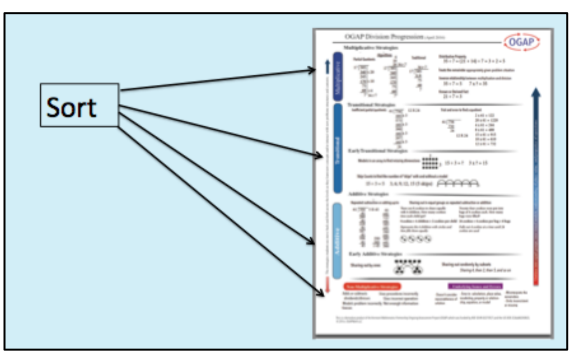
The Smith's have 3 windows on the front side of their house like the picture below. Each window contains many panes of glass.



How many panes of glass are there in the 3 windows on the front of the Smith's house?

Show your work.

Using the OGAP Quick Sort



1) What is the evidence of developing understanding?

2) What issues are evidenced in the student work?

7a- The Smith's Windows Case Study Handout

Recording Student Work

Item	Content (e.g., context, type of number)	Multiplicative	Transitional		Additive		Non-multiplicative Reasoning
			Transitional	Early	Additive	Early	

Underlying Issues or concerns								
Unreasonable	Misinterpret meaning of remainders	Place value error	Units inconsistent or absent	Property or relationship error	Calculation error	Equation error	Model error	Vocabulary error

Select and sequence the student work.

a) Select and sequence student work from the solutions you sorted to respond to the evidence in the student work.
b) Why did you choose these solutions?
c) Why did you sequence them in the order you did?
d) What questions would you ask students about the solutions to help move them along the multiplication progression?

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Equations	**Total**

Array Race to 100
Directions

Materials

2 six-sided dice 1-6
Game sheet per player
pencil

Goal

To be the first player to shade their entire 10 x 10 grid with arrays with no spaces left over.

Directions

1. Player 1 rolls the 2 dice. Player 1 records the multiplication fact and product in their equation chart. Player 1 draws, shades, and labels a rectangular array on the 10 x 10 grid that exactly matches the equation they just recorded in their equation chart. (ex: roll 6, 3 and write 6 x 3 = 18 in the equation chart. Shade an array with the dimensions 6x3 or 3x6 on the 10x10 grid. Label the array with the fact and product.)
2. Player 2 repeats the same steps.
3. Players continue in this way, taking turns and keeping a running total of the sum of the products.
4. Players continue in this way until one player has filled the entire 10x10 grid and all the equations match the grid and total 100.
5. If a player rolls a multiplication equation that cannot fit in the remaining space, the player loses a turn.

Variation: Instead of only shading the array that is an exact match to the factors rolled, players could use another set of factors for the same product that would fit the available area of their 10x10 grid.(ex: 8x1 could become a 2x4 because they both equal 8)

Variation: At first students should not use the distributive property of multiplication to fill the area but as they become more accomplished at understanding the relationship between the entire area and the related parts, they could use a distributive strategy to complete the 10 x 10 grid. (Ex: Roll 6 x4 and use it as 3 x 6 and 1 x 6.)

Variation: Change the size of the grid or the numbers on the dice.

Multiples of 4 Pathway
Directions

Materials

1 twelve-sided die 1-12
2 different colored pencils
1 game sheet for 2 players

Goal

To be the first player to connect a path from the start row to the finish row.

Directions

1. Player 1 rolls the die. They multiply the number rolled by 4. They state the fact and product orally. Player 1 selects the product from any row on the gameboard and circles it with their colored pencil.
2. Player 2 follows the same set of steps.
3. Play continues in this way, taking turns, until someone has connected a path from the start row to the finish row.
4. Players may jump around on the gameboard, choosing any number that corresponds to the product they roll. In the end they must connect a path from the top to the bottom row and some of the product/multiple they choose throughout the game may not be included in their final path.
5. If a product/multiple has been selected by player 1, it may not be selected by player 2, and vice versa.
6. If a player states the incorrect product, they lose a turn.

Variation: Pathways can be played with any numbers by creating new boards.

Multiples of 4 Pathway

Start

4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48
4	8	12	16	20	24	28	32	36	40	44	48

Finish

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

FACTORS

1 2 3 4 5 6 7 8 9

1. Player A places markers on 2 factors. They multiply the factors, find the product, and shade the box for that product.
2. Player B moves only 1 marker to another factor and shades in the product. (Both markers can go on the same number)
3. Play continues in this way until one player gets 4 in a row- vertically, horizontally and diagonally.

Remember: Only 1 marker can be moved at time.

Here I Am Multiplication

1. This game can be played with two or more players and is a math version of Battleship.

2. If more than 2 people play than one person becomes the leader and the others try to guess where the words are hidden. If only 2 players play then each one may hide words and guess each other's.

3. Hide the words HERE, I, and AM on the game board. The letters in a word must be in adjacent boxes vertically, horizontally, or diagonally but the words can be placed anywhere on the game board.

4. The game begins when one person tries to find a word by saying the multiplication problem with the correct answer that goes with a box they are trying to check on. If that box has a letter in it then the leader says "hit" and tells what the letter is and if there is no letter there then the leader says "miss" and writes the answer to the multiplication fact there.

5. Keep playing in this way until all the words on the board are found.

X	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

176 ÷ 12 = 14⅔

A. Fran's rug is 176 square feet. If one side of Fran's rectangular rug is 12 ft. what is the other dimension?

B. Penny is running in a relay race that is 176 km long. There are 12 runners who are running an equal amount of the race. How far is each runner running?

C. Bob has 176 oz. of maple syrup. Bob is bottling the syrup in 12 oz. containers. How many containers does he need to bottle all of the syrup?

D. The kids earned $176.00 at the carwash. If all 12 kids want to share the money equally, how much will they each get?

E. Stan has 176 inches of ribbon. Stan is making bows. Each bow needs 12 inches of ribbon. How many bows can she make?

F. Mary's bakery recipe for cookies requires 12 grams of baking soda. If Mary has 176 grams of baking soda, how many cookie recipes can she make?

G. Kyle has 176 eggs. Kyle is putting them in cartons of 12. How many cartons does he need?

H. Beth is making teams for field day. There are 176 students and 12 teams. How many students are on each team?

I. There are 176 cards in a math game that must all be dealt out. There are 12 players. How many cards does each person get?

J. A stretch of road on Route 17 is being paved. The stretch of road is 176 miles. If the crew can pave 12 miles a day, how long will it take them to finish the job?

K. Sue has 176 inches of string. How many feet of string does Sue have?

L. When fully stretched a rubber band is 176 inches long. When not stretched the rubber band is 12 inches long. When fully stretched, how many times longer is it than when not stretched?

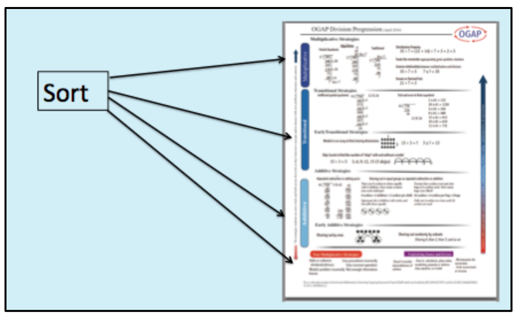
Lesson Goal: To use strategies based on place value and visual models to divide 2- and 3- digit numbers by 1-digit numbers.

Exit Card

1) Solve the task.
2) What are the structures of this exit task?
3) What errors might students make?
4) Why do you think Ms. Ward selected this question?

> The 4th grade class earned $132 from a bake sale.
>
> They decide to donate an equal amount of money to four different organizations.
>
> How much money will the class donate to each organization? Show your work.

Using the OGAP Quick Sort



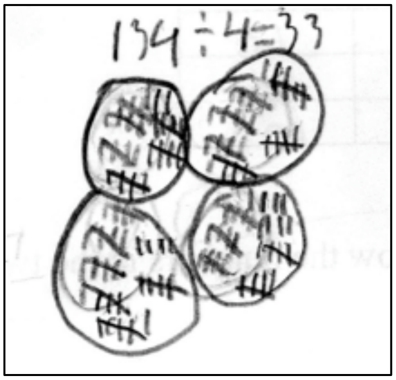
Item	Content (e.g., context, type of number)	Multiplicative	Transitional		Additive		Non-multiplicative Reasoning
			Transitional	Early	Additive	Early	

Underlying Issues or concerns								
Unreasonable	Misinterpret meaning of remainders	Place value error	Units inconsistent or absent	Property or relationship error	Calculation error	Equation error	Model error	Vocabulary error

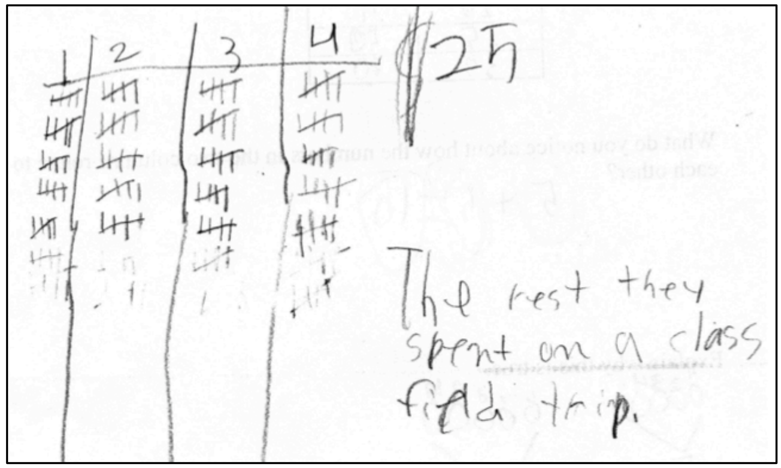
1) What is the evidence of developing understanding?
2) What issues are evidenced in the student work?
3) What should Ms. Ward do to respond to the work instructionally?

After sorting the work Ms. Ward was pleased to see that all the students recognized the task as division and had a variety of informal strategies to find the solution.

Like you, she could see that these students used a sharing out by 1s strategy while the rest of the class had more efficient methods relying on unitizing.



Student G



Student J

Since the lesson for the next day was focused on introducing partial quotients using a menu, she decided to work with these 2 students in a small group while the rest of the class worked on a set of mixed review problems.

Help Ms. Ward

1) What do these 2 students understand that can be built upon to move them along the progression?
2) What would be your instructional goal for these students?
3) Use your knowledge of problem structures to modify the task that could be used to work with these students to move them beyond a sharing out by 1s strategy
4) What questions would you ask to help them focus on using a "groups of" strategy to divide?

Modified Task:

Questions:

Concept/Property Item Analysis		
Item	**Does not apply or demonstrate understanding of targeted concept, relationship, or property**	**Demonstrates understanding of targeted concept, relationship, or property**

Underlying issues or concerns in student solutions:

______ Unreasonable

______ Misinterprets remainders

______Place value error

______ Units inconsistent or absent

______ Other

______ Property or relationship error

______ Calculation error

______ Equation error

______ Model error

______ Vocabulary error

Notes:

Items #	Content (e.g., context, type of number)	Multiplicative	Transitional		Additive		Non-multiplicative Reasoning
			Transitional	Early	Additive	Early	

Underlying Issues or concerns

Unreasonable	Misinterpret meaning of remainders	Place value error	Units inconsistent or absent	Property or relationship error	Calculation error	Equation error	Model error	Vocabulary error

Thoughts on Administering the OGAP Multiplicative Reasoning Pre-Assessment

An important component of the Ongoing Assessment Project involves gathering information about student understanding of multiplication and division concepts before beginning instruction through the administration of a pre-assessment. This pre-assessment is designed to elicit developing understandings, pre-conceptions, misconceptions, strategies that student use along the Progression, and common errors that students make when solving questions involving multiplication and division. It is designed to provide initial information that can inform planning. It is NOT, however, a complete assessment of all there is to know about your students' performance for multiplication and division at your grade level. It is in this spirit of formative assessment that we offer the following thoughts on administering the pre-assessment.

Tips for Students

Let the students know that this is a pre-assessment on material that they will be learning this year so some or all of the material may be new to them. Encourage them to try their best even if they are unsure. Remind them that the information will help you in your planning, and will not be used as a grade.

Time

The amount of time students need to complete the pre-assessment will differ depending on the grade level and the number of items in your pre-assessment. The pre-assessment can be administered in numerous ways. Some teachers choose to spread the assessment over several days while others administer the entire assessment in one class period. Again, the purpose is to collect evidence from your students so feel free to choose a schedule that works best for your students.

Level of Teacher Assistance

The purpose of formative assessment is to collect evidence that will help you best meet the needs of your students. With this in mind, feel free to read any items to students who you feel need this type of accommodation. You may also decide to scribe for students who require assistance with writing. Although no special materials are needed to complete the pre-assessment, students can use tools or manipulatives that are part of regular classroom instruction. By all means assist students in decoding non-mathematical vocabulary. *You should not, however, help students interpret any mathematics content.*

Final Thoughts

The ideas above are not intended to be used as a "checklist of do's and don'ts" but rather as a way to communicate the spirit in which the pre-assessments are best administered to your students. Please bring the completed pre-assessments to the December session. A major goal of these sessions is to help you learn how to analyze the evidence in your students' responses and use your findings to influence your upcoming proportionality instruction.

Name___________________________

Date _____________________________

1)

Look at this equation.

6 x 4 = 24

Draw a model that represents this equation.

E9(edited)

Name___________________________
Date _____________________________

2)

There are 12 cookies and 4 children. How many cookies will each child get?

Show your work.

EG44

3)

One tricycle has three wheels.

A. How many wheels do 4 tricycles have? Show your work.

B. How many wheels do 15 tricycles have? Show your work.

Name___________________________
Date _____________________________

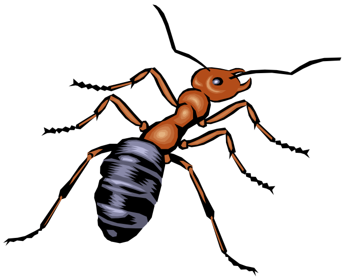
4)

Anna has a sheet of stickers. There are 4 rows of 9 stickers on the sheet of stickers.
How many stickers are there all together?

Show your work.

EG26

5)



There are 8 ants in an ant farm.

Each ant has 6 legs.

How many legs do the ants have all together?

EG62

.

Name__________________________

Date _____________________________

6)

Look at this equation.

10 x 2 = 20

Circle the story problem that can be solved using this equation?

A. Jill has 10 stickers and Bill gives her 2 more. How many does she have all together?

B. Jill has 10 stickers and Bill has 2 stickers. How many more stickers does Jill have?

C. Jill has 10 stickers and shares all her stickers with 2 friends. How many stickers do Jill and her friends get?

D. Jill has 10 sheets of stickers with 2 stickers on each sheet. How many does she have all together?

Explain your choice.

____

Grade 4 Name___________________
Multiplicative PreAssessment Date: ___________________

1)

Look at this equation.

$$\boxed{\mathbf{8 \times 4 = 32}}$$

Draw a model that represents this equation.

2)

A. Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there all together? Show your work.

B. John bought 12 boxes of crayons. Each box contained 64 crayons. How many crayons were there all together? Show your work.

Grade 4 Name__________________
Multiplicative PreAssessment Date: __________________

3)

An octopus has 8 legs. There are 96 legs in the aquarium.
How many octopuses are there in the aquarium?

4)

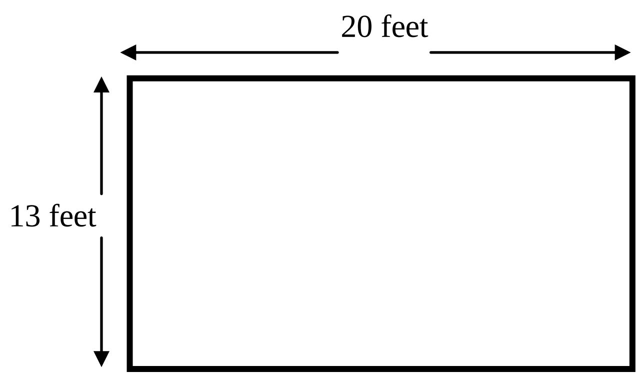
Katy and Lisa have a package that contains 50 feet of ribbon.
Four feet of ribbon is needed to make one bow.

Katy says they have enough ribbon to make 12 bows.
Lisa says they have enough ribbon to make 13 bows.

Who is correct? Explain your thinking?

Grade 4 Name__________________
Multiplicative PreAssessment Date: __________________

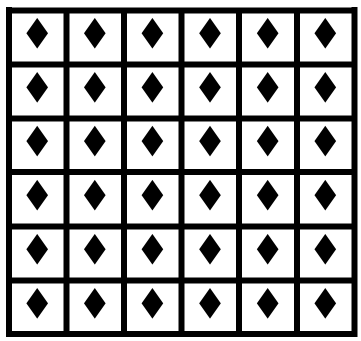
5)
Look at this diagram of Mark's bedroom floor.

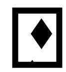


How many *one foot square tiles* are needed to tile the floor?

Show your work.

6) Here is a rectangle made with tiles.



 **Represents 1 tile**

A. Write a multiplication equation that can be used to find the total number of tiles.

B. Give another set of dimensions for a rectangle that has the same number of tiles.

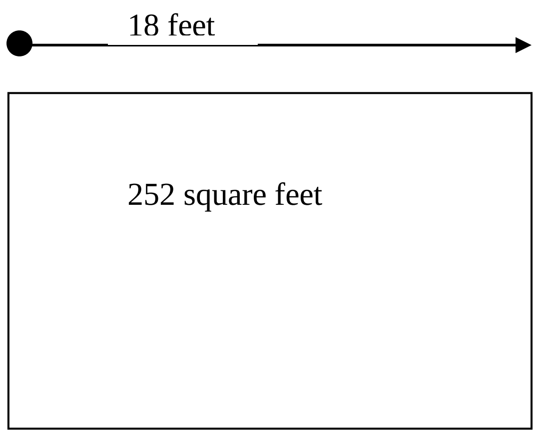
(EG19)

Grade 5
MR Pre-Assessment

Name: ________________________________
Date: ________________________________

1) (RA13)

Dean puts a fence around his garden. The area of his garden is 252 square feet. One side of his garden is 18 feet long. How long is the other side of the garden? Explain your thinking.



Grade 5 | Name: ______________________________
MR Pre-Assessment | Date: ______________________________

2) (EG13)

A. Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there all together? Show your work.

B. John bought 12 boxes of crayons. Each box contained 64 crayons. How many crayons were there all together? Show your work.

Name: ________________________________
Date: ________________________________

3)

The Champlain Music Festival will be held in an area where there is no parking. This means that all 2,881 participants will have to take a shuttle bus from the parking lot to the festival site.

If a shuttle bus can transport 67 riders each trip, how many trips will it take to get all the participants to the festival site?

Show your work.

 A derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project funded by The National Science Foundation (Award Number EHR-0227057) and the US Department of Education (S366A

Name: ______________________________
Date: ______________________________

4)

Noah's dad rides the bus to work 5 days a week. Below are two different ways he can buy his bus pass. Which is the better deal? Show your work.

One Day Pass **$2.50**	**Eight-Week Pass** **$85.00**

Name: ________________________________
Date: ________________________________

5)

Look at this equation.

$$\boxed{\mathbf{12 \times 6 = 72}}$$

Draw a model that represents this equation.

Name: ________________________________
Date: ________________________________

6) Solve each of the problems below.

A) Five friends earned 16 dollars shoveling snow. They want to share **all** of the money equally. How much money will each person get? Show your work.

B) There are 16 sixth graders. They need to form 5 teams for a special event. **All** of the sixth graders need to be on a team. How many sixth graders should be on each team? Show your work.

C) Five friends share 16 cookies equally. **ALL** the cookies will be shared by these friends. How many cookies will each friend get? Show your work.

Grade 2 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving addition and subtraction.**
- **Add and subtract within 20.**
- **Work with equal groups of objects to gain foundations for multiplication.**

Number and Operations in Base Ten

- **Understand place value.**
- **Use place value understanding and properties of operations to add and subtract.**

Measurement and Data

- **Measure and estimate lengths in standard units.**
- **Relate addition and subtraction to length.**
- **Work with time and money.**
- **Represent and interpret data.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

[1]See Glossary, Table 1.
[2]See standard 1.OA.6 for a list of mental strategies.
[3]Explanations may be supported by drawings or objects.

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

Geometry 2.G

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

[4]See Glossary, Table 1.
[5]Sizes are compared directly or visually, not compared by measuring.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving multiplication and division.**
- **Understand properties of multiplication and the relationship between multiplication and division.**
- **Multiply and divide within 100.**
- **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

Number and Operations in Base Ten

- **Use place value understanding and properties of operations to perform multi-digit arithmetic.**

Number and Operations—Fractions

- **Develop understanding of fractions as numbers.**

Measurement and Data

- **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7.

2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$.

3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If* $6 \times 4 = 24$ *is known, then* $4 \times 6 = 24$ *is also known. (Commutative property of multiplication.)* $3 \times 5 \times 2$ *can be found by* $3 \times 5 = 15$, *then* $15 \times 2 = 30$, *or by* $5 \times 2 = 10$, *then* $3 \times 10 = 30$. *(Associative property of multiplication.) Knowing that* $8 \times 5 = 40$ *and* $8 \times 2 = 16$, *one can find* 8×7 *as* $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. *(Distributive property.)*

6. Understand division as an unknown-factor problem. *For example, find* $32 \div 8$ *by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

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[1]See Glossary, Table 2.

[2]Students need not use formal terms for these properties.

[3]This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[5] 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
   b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

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[4]A range of algorithms may be used.
[5]Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

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[6]Excludes compound units such as cm^3 and finding the geometric volume of a container.

[7]Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

Geometry 3.G

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Grade 4 Overview

Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten[2] 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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[1]See Glossary, Table 2.
[2]Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions[3] 4.NF

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

[3]Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

   b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

[4]Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Grade 5 Overview

Operations and Algebraic Thinking

- **Write and interpret numerical expressions.**
- **Analyze patterns and relationships.**

Number and Operations in Base Ten

- **Understand the place value system.**
- **Perform operations with multi-digit whole numbers and with decimals to hundredths.**

Number and Operations—Fractions

- **Use equivalent fractions as a strategy to add and subtract fractions.**
- **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

Measurement and Data

- **Convert like measurement units within a given measurement system.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

Geometry

- **Graph points on the coordinate plane to solve real-world and mathematical problems.**
- **Classify two-dimensional figures into categories based on their properties.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Number and Operations in Base Ten 5.NBT

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number and Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
    a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
    b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5. Interpret multiplication as scaling (resizing), by:
    a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
    b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]
    a. Interpret division of a unit fraction by a non-zero whole number,

[1]Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

    a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

    b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

    a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

    b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

    c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
4. Classify two-dimensional figures in a hierarchy based on properties.

TABLE 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

Table 2. Common multiplication and division situations.[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

Table 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.